VOL. 23 No. 278

WIRELESS ENGINEER

NOVEMBER 1946.

Editor
W. T. COCKING M.I.E.E.

Managing Editor
HUGH S. POCOCK, M.I.E.E.

Technical Editor
Prof. G. W. O. HOWE, D.Sc., M.I.E.E.

Editorial Advisory Board.—F. M. COLEBROOK, B.Sc., A.C.G.I. (National Physical Laboratory), L. W. HAYES, O.B.E., M.I.E.E. (British Broadcasting Corporation), Professor E. B. MOULLIN, Sc.D., M.I.E.E., A. H. MUMFORD, O.B.E., B.Sc. (Eng.), M.I.E.E. (G.P.O. Engineering Department), R. L. SMITH-ROSE, D.Sc., Ph.D., M.I.E.E. (National Physical Laboratory).

EDITORIAL

Permeability of Iron-Dust Cores

IN the Editorial of January 1933 we considered this subject from what we thought at the time to be a new point of view; viz., by picturing the iron particles to be cubes arranged in a certain way. As we mentioned recently we subsequently found that this same assumption had been made by Doebke in 1930. In this current number we publish a letter from H. W. Lamson dealing with the subject but containing some misunderstandings which we wish to clear up. In the first place we must emphasize that we were considering practical iron-dust cores, that is, material in which the iron occupies the major part of the space, and we made approximations based on this hypothesis; in fact, the whole treatment assumed that the spaces between the cubes —the interstices as we called them—were relatively small. It is therefore not surprising if the formulae derived give quite erroneous results when applied to cases in which the iron only occupies a small fraction of the volume. In a covering letter Mr. Lamson says that he was "somewhat puzzled to note that, applying the condition of minute concentration of iron to your equation in terms of α, the value of μ approaches $1/\mu$, rather than unity as α approaches zero." Mr. Burgess also refers to this point in a letter published in this number. We were not concerned with minute concentrations of iron but with iron-dust cores, and the formulae developed were not intended to be applied to a dielectric containing a sparse sprinkling of iron particles.

The formulae that we derived, viz.

$$\mu = \frac{\mu_1 - \frac{2}{3}\delta(\mu_1 - 1)}{1 + \frac{1}{3}\delta(\mu_1 - 1)}$$

where μ_1 is the permeability of the iron particles and δ the small fraction of the total volume occupied by the insulating material, approximates to $\mu = \frac{3}{\delta} - 2$ as μ_1 is increased. As we showed in 1933, if μ_1 is infinitely great the formula gives $\mu = 28$ for $\delta = 0.1$ and 58 for $\delta = 0.05$.

Mr. Lamson is mistaken when he says that all three hypotheses are based on the assumption that the flux through the medium is in the form of parallel straight lines. The Poisson-Maxwell formula for spherical particles—to give it the name suggested by Mr. Fairweather in our August number—which was used by Mr. Burgess in June, makes no such assumption. It assumes that the flux is concentrated in the sphere just as it would be if no other sphere were near it. Maxwell* makes this quite clear and says "when the distance between the spheres is not great compared with their radii, then other terms enter into the result which we shall not now consider." The only one of the three hypotheses discussed by Mr. Lamson which ignores the concentration of flux in the iron particles in preference to the surrounding medium is the second one, which he himself

**Elec. & Mag.* Vol. 1, Section 314.

has suggested. With his so-called random alignment of cubes the flux density is the same everywhere, and it is not surprising that it gives results entirely different from those given by the other two hypotheses. It is reassuring to note that the other two hypotheses, although so fundamentally different, give results which differ so little for practical values of the space-factor.

The large discrepancies between these calculated values and the experimental results obtained by Legg and Given are very surprising but very satisfying from a practical point of view. It was rather a poor outlook for dust cores when it was thought that the permeability, even with the best material, could not exceed about 50. In a Special Report issued by the Radio Research Board in 1934 and published by the Stationery Office, entitled "Magnetic Materials at Radio Frequencies," it was stated that "all the effective permeability formulae . . . . agree in the main conclusion . . . that a figure of about 80 is the highest practically attainable, even with particles of the highest permeability." If the figures given by Legg and Given are reliable a dust-core permeability of over 150 can now be obtained with particles of which the permeability is only 220, but we must confess that we are not convinced that such a result can be obtained if the particles are properly insulated from each other. It could be obtained if the iron were linked up in the form of threads in the direction of the magnetic field. To take two extreme cases, laminations normal to the flux, i.e. in effect as in Lamson's hypothesis, would have a resultant permeability equal to $\mu_1/[\mu_1 - p\,(\mu_1 - 1)]$ which for $\mu_1 = 220$ and $p = 0.9$ gives a permeability of 9.6, whereas for the same quantity of iron, that is, the same value of p, arranged in the form of fine iron wires in the direction of the field, the resultant permeability would be simply $1 + p(\mu_1 - 1)$ which is equal to 198.1. Our formula for cubes gives 24 which seems to us to be more probable than the figure 128.3 given by Legg and Given unless the iron is largely in the form of fine threads in the direction of the field—a very desirable achievement.

G. W. O. H.

DOUBLE-DERIVED TERMINATIONS*

An Extension to Filter Theory

By R. O. Rowlands, B.Sc.

(The General Electric Co., Ltd., Coventry).

Introduction

IN an earlier paper[1] the author described a method of terminating complementary filters at their common ends in such a way that the real part of the image impedance at these terminals is that of a double-derived section.

When the impedance function is prescribed, however, the attenuation function of the terminating section is determined.

In the present paper the method is extended to allow more latitude in the choice of attenuation function when the impedance function is prescribed.

Composite Filters

A composite filter is built up as a ladder network by connecting a number of conventional derived-sections in tandem with the termination. Since the actual impedance looking in at one end of the filter depends upon the degree of matching at the other end, the filter will normally be terminated with a double-derived half-section at the end remote from the common terminals. The "m" values chosen for the intermediate sections are such that the composite filter gives the required attenuation in its stop range. A typical filter is shown in Fig. 1, $W'_{1^k m_2 m_1}$ being the impedance function whose real part is equal to $W_{1^k m_2 m_1}$.

The attenuation provided by the termination on the right-hand side is equivalent to that of a conventional section with m equal to m_2, while that provided by the termination on the left-hand side is that of two half-sections with m values equal to m_2, and m_1 times m_2. The filter, therefore, contains $1\frac{1}{2}$ sections with m equal to m_2. This is uneconomical. The problem is therefore how to get rid of a section with m equal to m_2

* MS accepted by the Editor, April 1946.

from the filter without affecting the impedance characteristic.

The formulae given in Appendix I enable us to interchange the m_2 and m_3 sections of the filter of Fig. 1. The m_2 section will now be adjacent to a conventional m_4 section, and so they can be interchanged by the and L_3 may, therefore, be taken out of the network and the remaining parts joined with perfect impedance matching at the junction. this means that a whole m_2-section has been removed from the network without affecting the latter's impedance characteristic at Terminals A and B. The composite filter

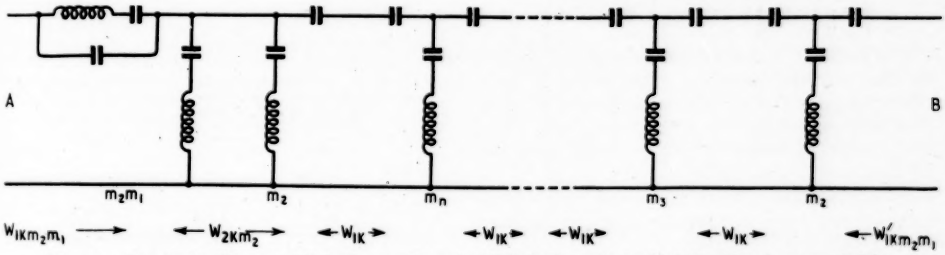


Fig. 1. *Composite filter; "A" is the remote end, "B" is the end at which the filter is connected in parallel with its complement.*

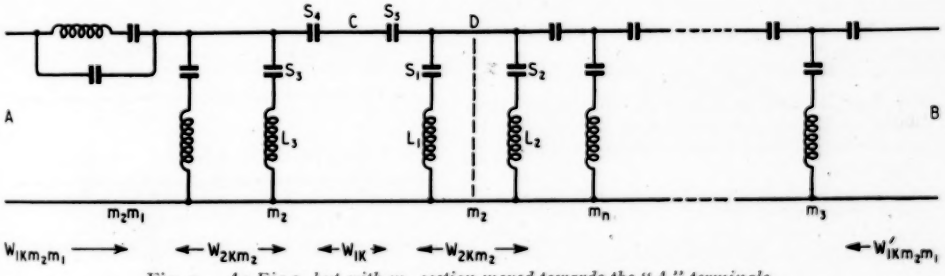


Fig. 2. *As Fig 1, but with m_2 section moved towards the "A" terminals.*

application of the same formulæ. By $n-2$ repeated applications the m_2 section can be moved so that it is adjacent to the conventional m_2 half-section preceding the double-derived half-section.

The filter will now be as shown in Fig. 2.

It will be seen that the coefficient of the component S_5 is "a," which is always equal to m_2, the value of this coefficient in a conventional section. If the transferred m_2 section, the shunt components of which are L and S, is split along the dotted line D into L_1 and S_1 in parallel with L_2 and S_2 in such a way that L_1 and S_1 are respectively equal to L_3 and S_3, then the image impedance of the half-section comprising S_5, S_1 and L_1 is W_{1k} at the junction C and W_{2km2} at the junction D. But W_{1k} was the image impedance at C before the network was split at D, so by the theorem proved in Appendix II the image impedances looking in both directions at the junction D are equal.

The section comprising L_1, S_1, S_5, S_4, S_3 may, therefore, be designed to have a prescribed double-derived impedance function but with latitude in the choice of the attenuation function of all but two half-sections which will have m values of m_2 and m_1 times m_2.

A filter made up of shunt-derived sections may be treated in the same way. The coefficients of the components "a," "b," etc., will in this case be the inverse of those for series-derived sections.

Practical Simplification

It may appear at first sight that in a filter with many sections the process of transferring the m_2 section from one end of the filter to the other is long and laborious. In practice this is not the case, for suppose q differs from its value in a conventional section by the factor δ;

$$\text{i.e., } q = \frac{1}{2m_2} + \delta$$

Then $$u = \frac{2m_3 + (m_3 - m_2)^2\,(1/2m_2 + \delta)}{(m_3 + m_2)^2}$$
$$= \frac{4m_2m_3 + (m_3 - m_2)^2}{2m_2(m_3 + m_2)^2} + \frac{(m_3 - m_2)^2\delta}{(m_3 + m_2)^2}$$
$$= \frac{1}{2m_2} + \left(\frac{m_3 - m_2}{m_3 + m_2}\right)^2\delta$$

The factor δ has been considerably reduced by the transformation. This value u now becomes q for the second transformation and so δ will be reduced a further amount. The best order in which to connect the sections of the composite filter of Fig. 1 so as to reduce δ, in the minimum number of transformations, to such a small value that it may be neglected is, to make

$$|m_3 - m_2| < |m_4 - m_2| < \text{etc.}$$

As soon as δ has reached this value the m_2 section may be removed from the filter without the necessity of any further transformation.

APPENDIX I

If the networks of Fig. 3 are equivalent with respect to their input and output terminals then we have the following transformation matrices.

For $\frac{1}{2}Z_{1k}$,

$$\begin{Vmatrix} a+b, & -b & , & o \\ -b, & b+c+d, & -d & \\ o & , -d & , & d+e \end{Vmatrix} = \begin{Vmatrix} 1 & x & o \\ o & y & o \\ o & z & 1 \end{Vmatrix} \cdot \begin{Vmatrix} f+g, & -g & , & o \\ -g & , & g+h+j, & -j \\ o & , & -j & , j+k \end{Vmatrix}.$$

$$\begin{Vmatrix} 1 & o & o \\ x & y & z \\ o & o & 1 \end{Vmatrix} = \begin{Vmatrix} f+(1-x)g & , & -g+x(g+h+j), & -xj \\ & - yg & , & y(g+h+j) , -yj \\ & - zg & , & z(g+h+j)-j , (-z+1)j+k \end{Vmatrix}.$$

$$\begin{Vmatrix} 1 & o & o \\ x & y & z \\ o & o & 1 \end{Vmatrix} = \begin{Vmatrix} f+(1-x^2)g+x^2(h+j) , & -yg+xy(g+h+j), \\ -yg+xy(g+h+j) & , y^2(g+h+j) \\ -zg+xz(g+h+j)-xj, & yz(g+h+j)-yj \end{Vmatrix}$$
$$\begin{matrix} , \\ -zg+zx(g+h+j)-xj \\ y(g+h+j)-yj \\ z^2(g+h)+(1-z)^2j+k \end{matrix} \Bigg\| \quad .. \quad .. \quad (1)$$

And for $2Z_{2k}$ substituting o for f, h, and k; p for g, and q for j,

$$\begin{Vmatrix} u-u & o \\ -u & u+v-v \\ o & -v-v \end{Vmatrix} = \begin{Vmatrix} (1-x)^2p+x^2q & -yp+yx(p+q) \\ -yp+xy(p+q) & y^2(p+q) \\ -zp+xz(p+q)-xq & yz(p+q)-yq \end{Vmatrix}$$
$$\begin{matrix} -zp+xz(p+q)-xq \\ y(p+q)-yq \\ z^2p+(1-z)^2q \end{matrix} \Bigg\| \quad .. \quad .. \quad (2)$$

Equation (2) Col. 2 gives

$$-u + (u+v) - v = -yp + yx\,(p+q) + y^2\,(p+q) + yz(p+q) - yq = 0$$

or $(p+q)(x+y+z) = (p+q)$

i.e., $$x + y + z = 1 \quad .. \quad .. \quad (3)$$

Equation (1) Col. 1 row 3 gives

$$-zg + xz(g+h+j) - xj = 0$$

or $$xz(g+h+j) = zg + xj \quad .. \quad .. \quad (4)$$

Equation (2) Col. 1 row 3 gives

$$-zp + xz(p+q) - xq = 0$$

or $$xz(p+q) = zp + xq \quad .. \quad .. \quad (5)$$

From (3), (4) and (5)

$$x = \frac{jp - gq}{jp - hq - gq} \quad ; \quad z = \frac{jp - gq}{jp + hp - gq}$$
$$y = 1 - (x + z)$$

If the network on the right-hand side of Fig. 3 is a conventional section joined to a double-derived terminating section then its components values will be as follows:—

$$f = m_3$$
$$g = \frac{1 - m_3^2}{2m_3}$$
$$h = m_2 + m_3$$
$$p = 1/(2m_3)$$
$$q = \frac{1 + m_1m_2}{m_2(1 + m_1)(1 + m_2)}$$
$$j = \frac{(1 - m_2)(1 + m_1m_2)}{m_2(1 + m_1)} = (1 - m_2^2)q$$
$$k = (1 + m_1m_2)$$

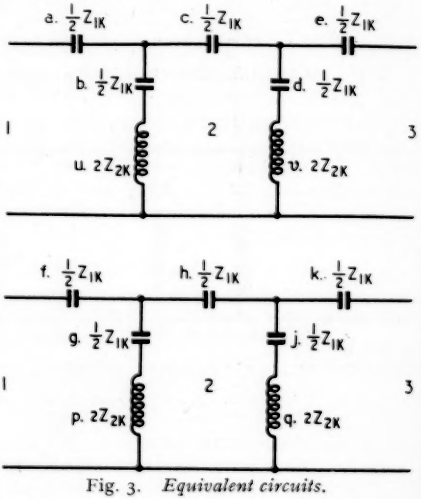


Fig. 3. *Equivalent circuits.*

And so

$$x = \frac{m_2 - m_3}{m_2 + m_3}$$
$$z = \frac{(m_3 - m_2)q}{(m_3 - m_2)q + 1}$$
$$y = \frac{(m_3 - m_2)^2q + 2m_3}{(m_3 + m_2)\{(m_3 - m_2)q + 1\}}$$

The component values of the network on the left-hand side of Fig. 3 are then given by

$$u = \frac{2m_3 + (m_3 - m_2)^2q}{(m_3 + m_2)^2}$$
$$v = \frac{2m_3 + (m_3 - m_2)^2q}{2m_3\{1 + (m_3 - m_2)q\}^2} \cdot q$$
$$a = m_2$$
$$b = \frac{(1 - m_2^2)\{2m_3 + (m_3 - m_2)^2q\}}{(m_3 + m_2)^2}$$
$$c = \frac{2m_3 + (m_3 - m_2)^2q}{1 + (m_3 - m_2)q}$$
$$d = \frac{(1 - m_3^2)\{2m_3 + (m_3 - m_2)^2q\}q}{2m_3\{1 + (m_2 - m_2)q\}^2}$$
$$e = k + \frac{(m_3^2 - m_2^2)q}{1 + (m_3 - m_2)q}$$

The above formulae are given in terms of m_2, m_3, q and k, and so they may be applied directly to obtain the first transformation. For the second and subsequent transformations, all that need be done is to substitute m_4 etc. for m_3 and use the appropriate values for q and k.

APPENDIX II

Theorem

If the four-terminal network N_{13} of Fig. 4 can be split into two cascade-connected four-terminal networks N_{12} and N_{23} in such a way that the image impedance at terminals 1, 1′ of both N_{12} and N_{13} is W_1, then the image impedances of both N_{12} and N_{23} at terminals 2, 2′ are equal.

Proof

Le the general circuit parameters of N_{12} and N_{23} in matrix form be $\begin{Vmatrix} A & B \\ C & D \end{Vmatrix}$ and $\begin{Vmatrix} a & b \\ c & d \end{Vmatrix}$ then the parameters of N_{13} are[2],

$$\begin{Vmatrix} A & B \\ C & D \end{Vmatrix} \times \begin{Vmatrix} a & b \\ c & d \end{Vmatrix} = \begin{Vmatrix} Aa + Bc & Ab + Bd \\ Ca + Dc & Cb + Dd \end{Vmatrix}$$

For the network N_{12}, $W_1^2 = \dfrac{AB}{CD}$

For the network N_{13}, $W_1^2 = \dfrac{(Aa + Bc)\ (Ab + Bd)}{(Ca + Dc)\ (Cb + Dd)}$

So $\dfrac{A\ B}{C\ D} = \dfrac{(Aa + Bc)(Ab + Bd)}{(Ca + Dc)(Cb + Dd)}$

i.e., $AB\{C^2ab + CD(ad + bc) + D^2cd\}$
$= CD\{A^2ab + AB(ad + bc) + B^2cd\}$

or $ACab(BC - AD) = BDcd(BC - AD)$

i.e., $\dfrac{ab}{cd} = \dfrac{BD}{AC}$

$W'_2 = W_2$

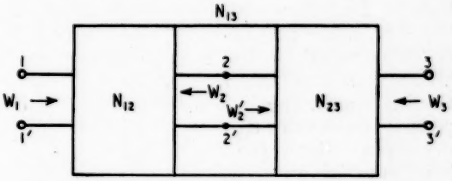


Fig. 4.

Acknowledgments

The author is grateful to the General Electric Co. for permission to publish this article, and to his colleague Mr. Li Wei Chen for helpful criticism.

REFERENCES

1 *Wireless Engineer*, Feb. 1946, the same notation will be used except that m and m_1 become m_2 and m_1 respectively in the present article.

2 See Guillemin's "Communication Networks," Vol. II p. 138, p. 145 and p. 171.

CHARACTERISTICS OF R.F. CABLES*

Determination of Impedance and Propagation Constants

By N. C. Stamford, M.Sc.Tech., and R. B. Quarmby, M.Sc.

(*Electrotechnics Department, Manchester University*)

Introduction

DURING the early stages of the war, and before more refined techniques, such as those described elsewhere by Dr. L. Essen, had been developed, the need arose for measurements at about 600 Mc/s of the phase and attenuation constants and the characteristic impedance of the Polythene insulated cables which were finding wide application in radiolocation equipments. For this purpose a technique was developed which was fundamental in principle and required only simple apparatus, and although it has now been superseded it would still seem to be advantageous for use in the instruction of students. The technique is equally applicable to coaxial- or twin-type cables and the frequency of 600 Mc/s happens to be a very convenient one at which to carry out the measurements.

* MS. accepted by the Editor, April 1946.

1. Measurement of the Phase Constant

The length of cable on which measurements are to be made is magnetically coupled to the oscillator at one end and left open at the other, as shown in Fig. 1. A thermojunction,

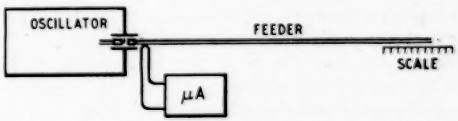


Fig. 1. *Arrangement of apparatus for the measurement of phase constant.*

the heater circuit of which is included in the coupling loop, enables the input current to the line to be measured. The procedure consists of cutting short lengths from the open end of the line and observing the input current for each value of line length. A curve of input current against line length is

plotted as the readings are taken, so that the size of the cut can be varied according to the steepness of the curve.

The input current is governed by the value of the input impedance of the line, namely $Z_0 \cdot \coth(\alpha + j\beta)l$, where Z_0 is the characteristic impedance of the line, α the attenuation constant, β the phase constant, and l the line length. This impedance passes through maxima for line lengths which are even multiples of one-quarter of a wavelength and through minima for line lengths which are odd multiples. The input current remains sensibly constant while the line impedance is low compared with the source impedance and falls sharply when the line impedance approaches a maximum value. The actual shape of the curve depends, of course, not only on the cable itself but also on the magnitude and phase of the coupling loop impedance. A typical curve is shown in Fig. 2.

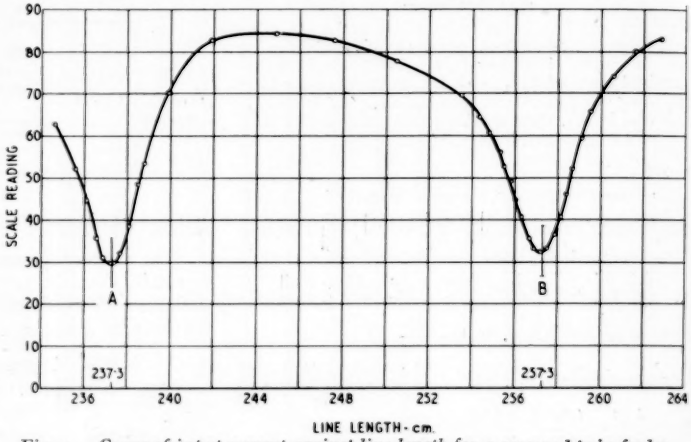


Fig. 2. *Curve of input current against line length for a screened twin feeder.*

Since the amount cut off between successive minima is a half-wavelength for the velocity of propagation in the line, it corresponds to a phase change of π radians. Thus if A and B are successive minima and x_1 is the distance in cm between them, the phase constant β is given by the relationship:

$$\beta = \pi/x_1 \text{ radians/cm} \quad .. \quad .. \quad (1)$$

In the curve of Fig. 2, for example, the minima occur at line lengths of 237.3 and 257.3 cm respectively. The distance x_1 is therefore 20.0 cm and the value of β is $\pi/20.0 = 0.157$ radian/cm.

Greater accuracy in the determination of the phase constant may be attained by making measurements over a number of half-wavelengths so that if the length of cable between $(n + 1)$ minima, having n maxima between them, is x_n cm the phase constant is:

$$\beta = n\pi/x_n \text{ radian/cm} \quad .. \quad .. \quad (2)$$

and an error in the location of one of the minima produces only $(1/n)$th of this error in the final answer.

This increase in accuracy can be obtained without cutting back the line over the full number of half-wavelengths for every sample of cable tested, if the equivalent length of the coupling loop is known. This may be determined by a trial experiment in which a length of the cable is cut back through a succession of minima until very short. Since it is then possible to predict with accuracy where the first minimum would come, and since this should be exactly at zero line length, it is easy to say what equivalent length of line must be attributed to the coupling loop. If this length is c cm and the $(n + 1)$th minimum occurs at a line length of l_n cm the value of x_n which must be used in equation (2) is $x_n = l_n + c$. The phase constant is then:

$$\beta = \frac{n\pi}{l_n + c} \text{ radian/cm} \quad .. \quad .. \quad (3)$$

Where there is any doubt as to the value of n, this may be found by cutting back the line until two minima are obtained, dividing the distance between them into the total line length (corrected for the equivalent length of the coupling loop), and taking n as the nearest integer.

Consider the curve of Fig. 2 again, in order to obtain a more accurate figure for β. A previous experiment had shown the equivalent length of the coupling loop to be 2.0 cm for this type of cable. The minima occur at line lengths of 237.3 and 257.3 cm which, when corrected, become 239.3 and 259.3 cm respectively. The separation between minima has already been seen to

be 20.0 cm, and dividing this into the corrected line lengths shows that these correspond to 12 and 13 half-wavelengths respectively. For the former, the value of β is thus $12\pi/239.3 = 0.1578_4$ radian/cm and for the latter, $\beta = 13\pi/259.3 = 0.1578_3$ radian/cm.

2. Measurement of the Attenuation Constant

The attenuation constant of the cable can be found by comparing the outputs of short and long lengths of line having equal inputs. To do this it is essential that the power measuring device be correctly matched to the line so that no standing wave exists along it. A thermojunction is used to measure the power output and is matched to the line through a "quarter-wave" transformer. If the line and thermojunction were both purely resistive, matching would be effected by a transformer having a characteristic impedance equal to the geometric mean of the line and thermojunction impedances, and a length of exactly a quarter-wave. Since, however, the thermojunction does not present a purely resistive load, the reactive component must be eliminated by alteration of the matching transformer length about the exact quarter-wavelength value. By successive adjustment of the characteristic impedance and length of the transformer an impedance match for both amplitude and phase is obtained.

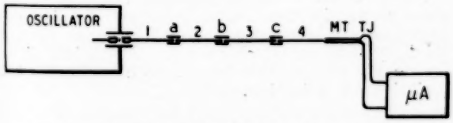


Fig. 3. *Arrangement of apparatus for attenuation measurements.*

Correct termination of the line is conveniently proved by observation of the output current as the line length is varied over a half-wavelength. The practical arrangement for doing this is shown in Fig. 3. The cable under test is coupled to the oscillator through a small coupling loop. Lengths 1, 2, 3 and 4 of the cable are linked together by sleeve-type couplings *a*, *b* and *c*. MT is the matching transformer between the end of the line and the thermojunction TJ, the response of which is measured on a micro-ammeter.

Of the four lengths of cable used, sections 1 and 4 are of arbitrary length, section 3 consists in the first instance of a short length L_1 and then of a long length L_2 by which means the attenuation of a length $(L_2 - L_1)$ of feeder is determined, and the length of section 2 is varied in steps over a range of at least three-quarters of a wavelength to test for the presence of a standing wave.

The couplings *a*, *b* and *c* consist of sections of brass tube into which the two ends of the cable to be joined are a push fit, so that the continuity of the outer conductor is maintained. The ends of the tube are split and provided with bushings which may be screwed along the split portions to increase their grip on the cable. Inside the tube is a spacer of the same dielectric material as that of the feeder to be tested, with drilled holes for the insertion of the inner conductors to be joined, small pools of mercury in the holes ensuring that a good contact is obtained. The form of the couplings for use with coaxial feeders is illustrated in Fig. 4.

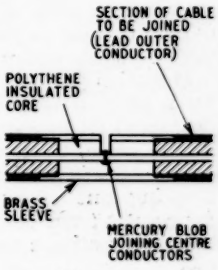

SECTION OF CABLE TO BE JOINED (LEAD OUTER CONDUCTOR)
POLYTHENE INSULATED CORE
BRASS SLEEVE
MERCURY BLOB JOINING CENTRE CONDUCTORS

Fig. 4. *Coupler for joining two lengths of coaxial feeder.*

The matching transformer MT consists of two parallel rods, each having an end cap of telescopic tubing so that the overall length of the transformer can be varied for phase matching. The rods are mounted so that the distance between them can be adjusted to vary the characteristic impedance.

The existence of a mismatch is observed by inserting a number of short sections of progressively increasing length between the couplings *a* and *b*, and plotting the output current against inserted length. The length and spacing of the matching transformer rods are successively adjusted and the test repeated until the output current variation is reduced to a minimum; ideally, to zero. The attenuation of a length $(L_2 - L_1)$ of the feeder line is then determined from the ratio of the currents indicated by the thermojunction when short and long lengths of cable respectively are inserted as section 3 between the couplings *b* and *c*. Thus, if the indicated currents are I_1 for the short length

of L_1 cm and I_2 for the long length of L_2 cm the attenuation constant is given by the relationship:

$$\alpha = \frac{20 \log_{10} (I_2/I_1)}{(L_2 - L_1)} \text{ decibels/cm}$$

3. Measurement of Characteristic Impedance

The characteristic impedance of the cable is determined using a simple variation of the arrangement used for the measurement of attenuation. In this connection, it has already been explained how, when all four sections of the line are of the same characteristic impedance, the thermojunction can be matched to give a substantially constant output current as the length of the line is varied incrementally in section 2, Fig. 3.

If section 3 of the system is now replaced by a quarter-wavelength of line having a characteristic impedance different from that of the cable, then, due to the transformer action of this section, the line beyond the coupling *b* no longer satisfies the conditions for correct termination and variation of the length of section 2 produces variations in the output current.

The first step in the practical procedure is to form all sections of the line from the cable under test and to adjust the matching of the thermojunction as for an attenuation measurement.

Section 3 is then replaced by a quarter-wavelength device of which the characteristic impedance can be varied systematically, and for each value of this characteristic impedance the output current variation is observed as the length of section 2 is changed. For one value of the characteristic impedance there will be no output current variation for small changes in the length of section 2 and this gives the desired cable impedance.

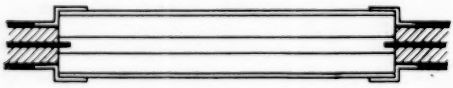


Fig. 5. *Air-spaced coaxial line with detachable end pieces and centre conductor of variable diameter.*

For measurements on coaxial cables a suitable form for this quarter-wavelength device is obtained by using air-spaced coaxial tubes, the impedance variation being effected by changing the diameter of the centre conductor. Satisfactory attachment to the test cable can be made by means of end pieces in the manner shown in Fig. 5. For twin feeders, quarter-wavelength lines having variable spacing may be used.

Acknowledgment

The technique described was developed during 1940-41 in the Electrotechnics Laboratories of the Victoria University of Manchester in a Research Group under the direction of Professor Willis Jackson to whom due acknowledgment is made.

BOOKS RECEIVED

Radio Valve Vade-Mecum, 1946, 6th Edition.

Pp. 232 + xii. Published by P. H. Brans, 28, rue du Prince Leopold, Anvers (Borgerhout), Belgium. Distributed by Ritchie Vincent & Telford, Ltd., Harrow, Middx. Price 12s. 6d. (Postage paid.)

This is a new edition of international valve data which has been extended by the inclusion of German and Italian military types of valves. Valve base connections are included as well as characteristics.

The presentation is in tabular form and the explanatory pages in English.

Quality Through Statistics (2nd Edition).

By A. S. Wharton, F.S.S., A.I.I.A. Pp. 62 + iv. Published by Philips Lamps, Ltd., Century House, Shaftesbury Avenue, London, W.C.2. Price 6s.

Demonstrations of Radio Aids to Civil Aviation.

Pp. 80+vii. Published by H.M. Stationery Office, York House, Kingsway, London, W.C.2. Price 5s.

Short technical descriptions are given of equipment shown in the United Kingdom by the Research and Development Establishments of the Ministry of Supply during the demonstrations given on behalf of the Provisional International Civil Aviation Organization.

Institute of Physics

A MEETING of the Electronics Group of the Institute of Physics will be held on Friday, November 8th, 1946, at 7 p.m., at the Lecture Theatre, New Physics Building, The University, Oxford Road, Manchester.

The subject will be "Contact Potentials," by Dr. F. A. Vick.

CORRECTION

Two errors occurred in the advertisement pages of *Wireless Engineer* for October 1946. Inadvertently, the price of the M.I.P. Series 100 Multi-Range Test Set advertised by Measuring Instruments (Pullin) Ltd., was given as £8, instead of the correct figure of £8 10s. 0d.

The 15in Goodmans Industries Ltd. loudspeaker advertised in the same issue was quoted as Type T2/1205/15. This should have read T10/1501/15.

NEGATIVE RESISTANCE CIRCUIT ELEMENT*

The Dynatron at High Radio Frequencies

By G. A. Hay, M.Sc., A.M.Brit.I.R.E.

(The Radiotherapy Centre at Leeds)

Introduction

THE use of the dynatron as a means of measuring audio- and low radio-frequency resistance is well known, and has been developed by several workers[3], [10], [14], [15], [16]. Apparently, however, the usefulness of this method has not been extended to the higher frequencies, although some work has been done with the dynatron at frequencies up to 25 Mc/s. The ease with which measurements of the dynamic resistance of tuned circuits may be made suggests that the dynatron would be a very useful tool at the higher radio frequencies if more were known about its properties in those regions. The work described below was carried out in an attempt to make the dynatron available for use at frequencies up to 100 Mc/s.

1. Review of Previous Work

The earliest applications of the dynatron at high radio frequencies lay in its use as an oscillator rather than a measuring device. In 1925 Gill and Morrell[6] constructed an oscillator using a tungsten-filament triode, the control grid having a high positive potential applied to it. The oscillator made use of secondary emission from the anode, but was not a true dynatron, as the grid was not held at a constant potential. In this circuit, the finite transit time of the electrons was said to give rise to irregularities above 10 Mc/s, but it appears unwise to draw any rigid conclusions from this work, in view of the unorthodox nature of the valve and circuit.

In 1931 Colebrook[1] used a tetrode as dynatron with a resonant circuit tuned to 20 Mc/s. Calculation showed that the zero-frequency differential resistance of the valve was numerically much smaller than the positive dynamic resistance of the tuned circuit, thus indicating an abnormal effect. No explanation was then attempted.

At about this time the dynatron was used as a measuring instrument by Iinuma[10], who had previously compared its results at frequencies up to 8 Mc/s with those given by other methods, and had found close coincidence. It was then used for measurements up to 25 Mc/s, but no adequate comparisons seem to have been made at these frequencies, beyond a few measurements using the series-resistance variation method, from which agreement was found with a tolerance of 10%. This author neglects dielectric losses in the valve, and although they may have been small in the particular specimen used, they have been found very important in the present work, being of the same order of magnitude as the negative resistance itself at 50 Mc/s or more. The results, however, suggest the type of behaviour to be expected, and have been fully confirmed in the present work.

In 1938 the operation of a triode dynatron as a generator of ultra-short waves was investigated by Meinke[11,12]. In this work the space current was used as a measure of negative resistance (as negative conductance is proportional to space current). Irregularities in behaviour were found at frequencies of the order of 30 to 50 Mc/s, and these were attributed to transit time effects. It is interesting to note that this author has fully realized the necessity for chokes isolating the r.f. circuits from power supplies, a precaution which was found essential in the present work. He also used large bypass capacitors built into the valve so that all electrodes except the anode should be truly at cathode potential for radio-frequencies.

Finally, the nature of dynatron negative impedance was investigated in 1943 by Chakravarti and Das.[2] Here the dynatron was regarded as an unknown impedance, and the magnitude of its resistance component found by a triple current method, using thermocouple meters and a standard resistance. The difficulties and errors in such a method must have been great, and this is

* MS accepted by the Editor, April 1946.

reflected in the results obtained, as in one specimen series of measurements, the numerical value of the negative resistance decreased irregularly from 15 kΩ at 3 Mc/s to a few hundred ohms at 30 Mc/s and then went actually positive at 40 Mc/s. The writer feels that more precise information is necessary to substantiate the results claimed from this work.

Although no further practical work on the dynatron appears to have been published, a theoretical analysis appeared in 1938 by Viti.[18] In this is emphasized the change in conductance due to dielectric losses in the valve, and also the fact that the interelectrode capacitance forms part of the tuned circuit and may change with frequency. Secondly, the effects of lead inductance are analysed, and it is concluded that the most important of these is the feeding back to the control grid of an appreciable fraction of the anode voltage. This occurs because the grid cannot be bypassed completely to cathode because of the grid-lead inductance, and so it acquires an oscillatory potential relative to cathode, which then varies the negative resistance periodically. From an analysis of this effect the author deduces that if the series-resonant frequency of the grid circuit is of the order of 300 Mc/s, the valve will be usable as a measuring instrument up to about 150 Mc/s. This, of course, disregards transit time effects, and attributes any loss in performance to circuit limitations only.

2. Optimum Working Conditions

The measurement of dynamic resistance with the dynatron at low radio frequencies has been fully treated elsewhere[14,15,16], but there are some facts which are not generally appreciated; these are enumerated below.

The process of measurement consists of two parts: (*a*) the neutralizing of a positive dynamic resistance by the negative differential resistance of the valve, and (*b*) the measurement of the negative resistance by a bridge method at 1,000 c/s, which may safely be assumed equal to the zero-frequency value. One must first choose a suitable working point. It is necessary that the growth of oscillation should be limited to a small value, less than one volt peak, by correct choice of the shape of the characteristic, and this will occur at a point of inflexion in the $I_a - V_a$ curve where the gradient of the curve, $\frac{dI_a}{dV_a}$, is a negative maximum.[15]

Most tetrodes and pentodes have a convenient knee in the characteristic at an anode voltage of about 10 volts (Fig. 1), and the proximity of the first bend limits the amplitude to a small value. In addition, the slope of the curve at this point is large, giving a satisfactorily small value of negative resistance for neutralizing bad tuned circuits. Further, it is necessary that the amplitude of the 1,000-c/s voltage applied from the negative-resistance bridge in (*b*) should be approximately the same as the r.f. signal, as if it were to sweep over a different length of curve the average negative resistance would be different.

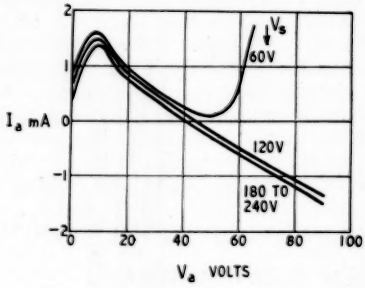


Fig. 1. *Static characteristics of AC/SG for a cathode current of 10 mA.*

In practice such resistance measurements are complicated by two other effects: (1) the anode–earth capacitance C_{ae}, and (2) the dielectric losses due to glass bulb, mica supports, etc. The equivalent circuit is shown in Fig. 2, and it is important to note that R_{diel}, representing the dielectric losses, will vary with frequency. Its value must therefore be known at all frequencies. At normal frequencies the anode–earth capacitance will be unimportant, but at higher frequencies it becomes comparable with the main circuit capacitance and its presence will be troublesome. Moreover, the effective anode–earth capacitance looking into the dynatron terminals will vary at the higher frequencies due to the presence of lead inductance in the valve.

The most useful valve for measurement purposes is the tetrode, although a pentode may be used with suppressor grid connected to screen. The optimum operating conditions have been discussed by the present

writer for pentodes,[8] and broadly speaking these remarks apply equally to pentodes and tetrodes, although the behaviour of a pentode at the higher radio frequencies may be abnormal due to the larger anode–screen distance and larger electron transit time.

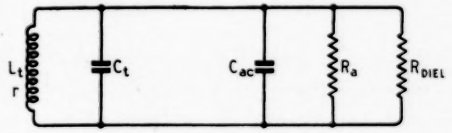


Fig. 2. *Simple equivalent circuit of the dynatron.*

2. Dynatron at High Radio Frequencies.

An approximate calculation of the electron transit time between anode and screen of a conventional tetrode gives a value in the region of 3×10^{-10} sec. This alone can hardly affect the operation of the valve at 100 Mc/s, and, moreover, the dynatron presents least opportunity of all the oscillators for space charges and other interfering factors to appear. It seems possible, therefore, that the abnormalities reported by previous workers might be explained in other ways.

Several methods were available for measurement of the negative resistance of the dynatron at high radio frequencies, but finally the method was chosen of using it to neutralize the positive dynamic resistance of a tuned circuit, which could be measured by one of the standard methods. This approximates most nearly to the conditions under which the valve will be used, and with obvious modifications has been used by Harvey in measurements on magnetron oscillators.[7]

Thus the negative resistance of the valve, R_a, at the working frequency could be found from the dynamic resistance R_d, and if the dynatron is behaving normally this should be equal to the zero-frequency resistance R°_a. Hence the ratio $\frac{R^\circ_a}{R_a} = \epsilon$ could be used as a measure of the behaviour of the valve, as any departure from unity would indicate some abnormal effect. It is also necessary to know separately the value of R_{diel} (the dielectric-loss resistance), as the value of this at the working frequency is not the same as at zero frequency. This was measured by connecting the valve across the tuned circuit with the cathode cold.

The negative resistance at zero frequency was measured by a negative-resistance bridge working at 1,000 c/s. The bridge circuit is shown in Fig. 3. Two direct-reading ranges were provided, reading 0 to 5,000 ohms and 0 to 35,000 ohms, and the accuracy of the bridge was checked by rearranging it to measure a known positive resistance. The self-capacitance of the dynatron was compensated by a balancing capacitor as described by Terman[16]. The bridge was fed from a 1,000-c/s low-distortion oscillator, with screened transformers in the generator and detector circuits. The detector comprized a high gain semi-aperiodic amplifier feeding headphones, and the final accuracy of measurement was estimated at $\pm$ 2%.

The dynamic resistance of the tuned circuit was measured at various frequencies by the capacitance-variation method, using a calibrated low-loss screened capacitor with a usable capacitance variation of 4 pF.

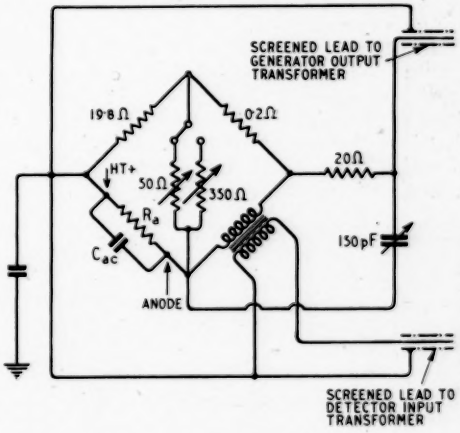


Fig. 3. *Negative-resistance bridge used for measurement of* R°_a.

The tuned circuits were built in a conventional manner, and as at the lower frequencies their dynamic resistance was off the range of the negative-resistance bridge, they were shunted by small metallized resistors as required. A voltage was induced in them from a medium-power oscillator very loosely coupled to the circuit to avoid interaction. Extensive screening was not available, and the whole apparatus was placed on an earthed aluminium sheet, and extension handles fitted where necessary.

The complete apparatus for the range 1 to 100 Mc/s is shown in Fig. 4, the valve used being a Mazda AC/SG. The only unconventional feature was the anode-voltage supply, which was designed for smooth variation between 7.5 and 13.5 volts and a very low anode-circuit resistance. The valve was decapped, and all electrodes but the anode were bypassed to cathode by 0.01-μF mica capacitors. The circuit had to be used in two conditions: (1) with the dynatron connected to a tuned circuit for the measurement of R_a, and (2) with the dynatron connected to a negative-resistance bridge for the measurement of R°_a. This changeover was made by crocodile clips to minimize the capacitive effects of switches. The valve voltmeter used for the dynamic-resistance measurement, and for detecting and adjusting oscillation amplitude, was connected permanently across the tuned circuit, as its losses were of no interest. For frequencies up to 50 Mc/s it consisted of a conventional square-law voltmeter using an acorn triode.

Measurements were made at first from 1 to 25 Mc/s, and the values of ϵ thus obtained are shown in Fig. 5. It departs little from unity up to 10 Mc/s, but after that irregular changes occur, the significance of which was not at first clear. However, it was noticed that these irregularities were closely related to large increases in tuned circuit losses, and the cause of this was finally traced by a process of elimination to series resonances in power-supply leads. When r.f. chokes had been added (at the points marked X in Fig. 4) to isolate the tuned circuit from everything except the

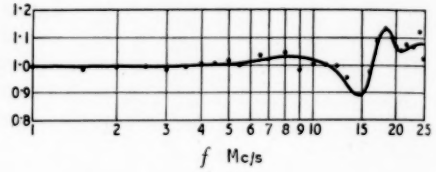


Fig. 5. *First measured values of ϵ; 1 to 25 Mc/s.*

dynatron and valve voltmeter, the irregularities disappeared completely. Results from 1 to 50 Mc/s are shown in Fig. 6, and values of R_{diel} in Fig. 7. (Inaccuracies in the values for R_{diel} at the lower frequencies are due to the expression for R_{diel} containing the small difference between two large quantities.)

At this point it was thought desirable to confirm the validity of ϵ as a measure of the dynatron's performance. The variation of the ratio with a change of R_d was therefore examined at a fixed frequency of 15 Mc/s.

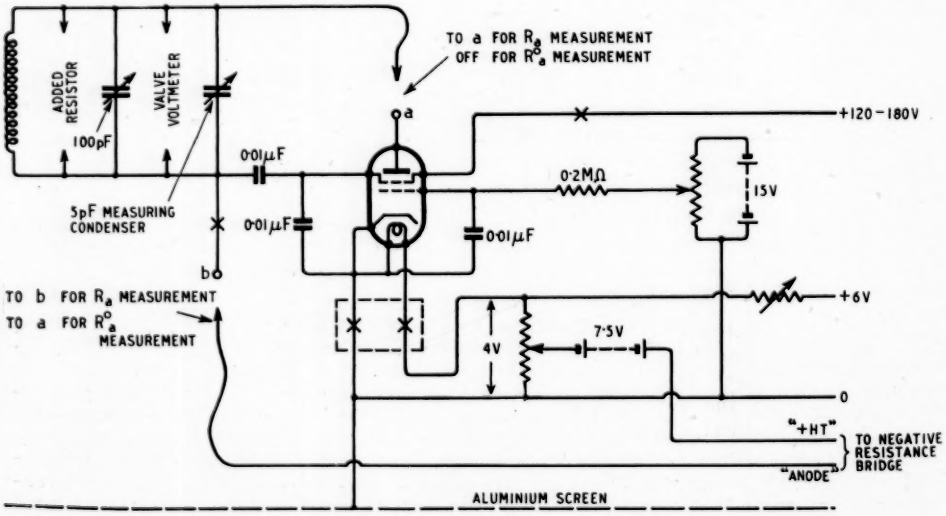


Fig. 4. *Circuit used for measurements: crosses denote the positions of single or double r.f. chokes added later.*

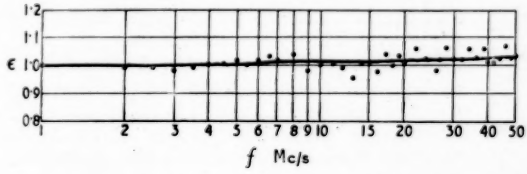
The results are shown in the Table, and confirm that ϵ does not vary appreciably for quite large changes in R_d.

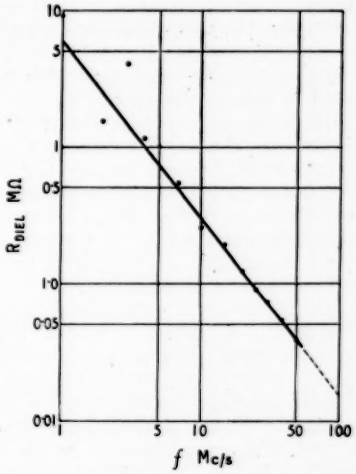
Shunt resistance (nominal) (kΩ)	$-R^{\circ}_{a}$ (kΩ)	$R_a = R_d$ (kΩ)	ϵ
∞	29.0	28.6	1.01
60	19.3	19.3	1.00
40	17.1	16.6	1.03
20	12.3	12.1	1.02
10	9.0	9.1	0.99

Variation of ϵ with changing R_a; frequency constant at 15 Mc/s.

Fig. 6 (*below*). *Final measured values of* ϵ *; 1 to 50 Mc/s.*

Fig. 7 (*right*). *Values obtained for* R_{diel} *up to 50 Mc/s.*





Above 50 Mc/s an acorn-diode voltmeter with a d.c. amplifier was substituted for the square-law meter, and an improved form of local oscillator was used. It was originally intended to use quarter-wave resonant lines at frequencies above 50 Mc/s, but the losses in the AC/SG at these frequencies were so great that the large resonant impedance of the line conferred no advantage. In addition, the anode–earth capacitance was about 10 pF, and this loaded the line excessively, so that most of it was inside the valve. So the apparatus was rebuilt with conventional tuned circuits of very low-loss construction, and all components grouped closely around the valve. Care was taken to make the anode lead as short as possible, so that its inductance could be neglected in subsequent calculations.

The results obtained were uniformly good, and are shown in Fig. 8. As it was suspected that irregularities might now be due to series resonance in the anode circuit of the dynatron, a rough measurement of this frequency was made and found to be 125 Mc/s. It was quite apparent at this point that any further attempts to increase the frequency would be unsuccessful, as the inductance of the external circuit was at its practical minimum, and the real obstacles lay in the lead inductance and inter-electrode capacitance of the valve. For still higher frequencies a special valve would be necessary, and it is hoped that this may form the basis of future work.

4. Discussion of Results

Examination of the results given in Fig. 6 shows that if the negative resistance of the AC/SG be measured at zero frequency or at 1,000 c/s, then it will be the same at all frequencies up to 50 Mc/s within a tolerance

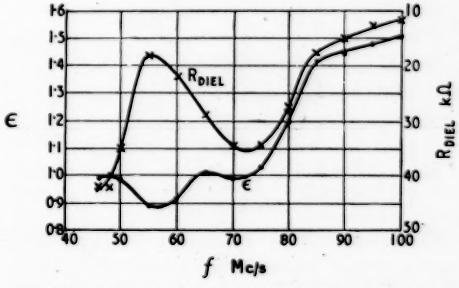


Fig. 8. *Values of* ϵ *and* R_{diel} *from 50 to 100 Mc/s.*

of $\pm$ 5%; i.e., ϵ lies between 0.95 and 1.05. No doubt measurements with more refined apparatus would confirm this result to a higher degree of accuracy. This conclusion is true only if the valve and associated r.f. circuits are completely isolated from power supplies, etc., by adequate chokes and bypass capacitors. This is a most important

point, and it seems to afford a ready explanation for the irregular results obtained by previous workers.

Above 50 Mc/s irregularities appear in the value of ϵ, (Fig. 8), and it was at first supposed that transit time effects were becoming apparent. Measurement of dielectric losses in the valve, however, showed that these irregularities were exactly coincident with sudden increases in energy absorption by the valve. The first of these occurs at about 56 Mc/s, and it is seen that a decrease in ϵ results. On the other hand, the next large decrease in R_{diel} occurs towards 100 Mc/s and higher, and this results in an increase in ϵ. It is unlikely that electron transit time is responsible for this, and apparently we must look to circuit properties for the explanation.

This idea has been established for triode oscillators by Gavin[5], who states that for triodes of normal construction, limitation of the maximum frequency is first brought about by circuits conditions rather than by transit-time effects. It has already been shown in Sect. 3 that the transit time in an AC/SG is roughly equal to one cycle of a 3,000-Mc/s oscillation, and therefore should not materially influence 100-Mc/s operation.

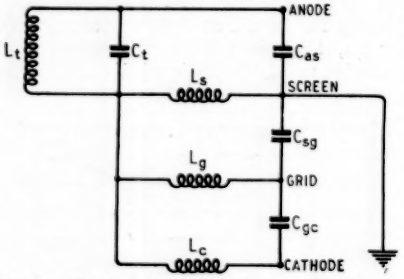


Fig. 9. *Equivalent circuit of dynatron showing inter-electrode capacitances and lead inductances.*

Considering now those circuit conditions which might affect the operation, a much more satisfactory explanation can be advanced. Fig. 9 shows the equivalent circuit of the dynatron, including inter-electrode capacitances and lead inductances, and considering r.f. conditions only. We may assume that the screen is at earth potential, and ignore the anode–grid capacitance. As the anode lead was short and taken to a top cap, its inductance has been neglected, and no account has been taken of mutual inductances and lead resistances.

In the anode circuit, the tuned circuit proper L_t and C_t has in parallel with it a series-resonant circuit L_s and C_{as}. Actually L_s is itself shunted by circuits consisting of L_g, C_{sg} and L_c, C_{gc}, but for simplicity we may assume all these to be lumped together to give an effective screen inductance L'_s.

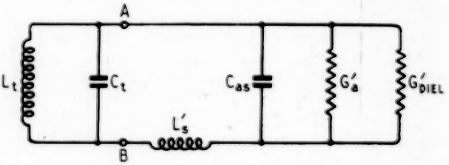


Fig. 10. *Equivalent circuit of dynatron showing effective screen lead inductance.*

This can now be calculated, for the resonant frequency of L'_s and C_{as} is 125 Mc/s (Sect. 3) and C_{as} is about 10 pF (makers' figure), so L'_s is 0.16 μH. By assuming $L_s=L_g=L_c=L$, and $C_{as}=C_{sg}=C_{gc}=C$ (a reasonable assumption from the geometry of the valve) we may calculate L, which is about 0.05 μH. This is the inductance of one of the leads from the base of the valve to the electrodes. By applying the dimensions of such a lead in a chart given by Gavin for the inductance of a straight wire,[5] the theoretical value is found to be 0.08 μH. In a similar manner the external inductance L_t is found to have a value of 0.09 μH, and the calculated figure for the coil alone is 0.05 μH. This neglects connecting leads, which would probably bring the inductance nearer to the experimental value. The agreement between these figures is not close, but in view of the approximations made it is considered to support the correctness of the equivalent circuit used.

In Fig. 10 the equivalent circuit is shown in another form, with

$$G'_a = \frac{1}{R'_a} \quad \text{and} \quad G'_{diel} = \frac{1}{R'_{diel}}.$$

These are the true values of negative conductance and dielectric loss, considered as measured across the anode–screen space, and the measured values (across AB) will only coincide with these if the reactances of L'_s and C_{as} are small and large respectively at the operating frequency, that is, at low and medium radio frequencies. If, however, these quantities become important in the circuit, the values of negative conductance and dielectric loss measured at the working frequency will be the effect of G'_a and G'_{diel} across AB; i.e., G_a and G_{diel}.

For convenience in manipulation, let us consider $G_a + G_{diel} = G$, and $G'_a + G'_{diel} = G'$. At an angular frequency ω, the parallel admittance of the anode–screen space is $(G + j\omega C_{as})$ and the total admittance of the valve between A and B is

$$\frac{(G' + j\omega C_{as})\ 1/j\omega L'_s}{G' + j\omega C_{as} + 1/j\omega L'_s}$$

Simplifying and rationalizing we have

$$Y_{AB} = \frac{G'}{(1 - \omega^2 L'_s C_{as})^2 + \omega^2 L'^2_s G'^2} - j\omega \frac{L'_s G'^2 + C_{as}(1 - \omega^2 L'_s C_{as})}{(1 - \omega^2 L'_s C_{as})^2 + \omega^2 L'^2_s G'^2}$$

At the moment we are mainly interested in the first term, for this gives the value of G for any angular frequency ω. Suppose further that ω_0 is the resonant frequency of L'_s and C_{as}, then $\omega_0^2 L'_s C_{as} = 1$. Hence

$$G = G'. \frac{1}{(1 - \omega^2/\omega_0^2)^2 + \omega^2 L'^2_s G'^2};$$

but $\omega^2 L'^2_s G'^2$ is small,

$$\therefore\ G = k\,.\,G' \quad \text{where} \quad k = \frac{1}{(1 - \omega^2/\omega_0^2)^2}$$

$$\therefore\ G_a + G_{diel} = k.(G'_a + G'_{diel})$$

$$\therefore\ G_a = k\,.\,G'_a, \quad \text{and} \quad G_{diel} = k\,.\,G'_{diel}$$

Now G'_{diel} may be estimated by extrapolation from Fig. 7, and if k were known at any frequency, the apparent G_{diel} could be calculated. But $k = G_a/G'_a = R^{\circ}_a/R_a = \epsilon$ hence $G_{diel} = \epsilon\,.\,G'_{diel}$, and this calculated value is compared with the measured value (obtained from Fig. 8) in Fig. 11. From 75 Mc/s upwards, although numerical agreement is absent, the curves follow exactly the same trend, and it is apparent that some such mechanism as outlined above must be responsible for the behaviour observed. Below 75 Mc/s, however, a sharp peak in G_{diel} occurs, and to explain this we must have recourse to a theory advanced by Viti (Sect. 1) in which the grid is said to acquire an oscillatory potential relative to the cathode. A positive increase of grid voltage causes an increase in anode voltage, and so if the feed-back potential is out of phase with the anode voltage a negative feed-back effect will be produced and the apparent G_a will be less. Hence the ratio $\epsilon = \dfrac{G_a}{G'_a}$ will be less than unity. It is quite impossible to calculate the magnitude of this effect, as so much may depend on stray inductive and capacitive couplings in the circuit and valve, but it may be assumed that the oscillatory control-grid voltage arises because of series resonance in the grid–cathode circuit, and the resonant frequency would appear to be about 55 Mc/s.

5. Conclusion

It now seems clear that any limitations in the behaviour of the dynatron up to 100 Mc/s

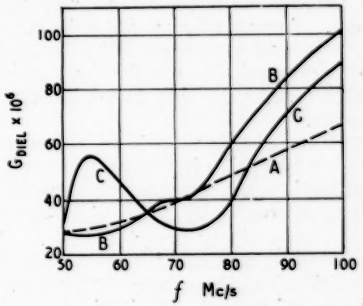


Fig. 11. *Comparison of measured values of G_{diel} with values obtained from theory: curve* A, *G'_{diel} obtained by extrapolation from Fig. 7; curve* B, *G_{diel} calculated from G'_{diel} and ϵ; curve* C, *G_{diel} measured.*

are not due to transit time effects, but merely to circuit conditions associated with the construction of the valve. Irregularities in behaviour may also arise as a result of resonance in power supply leads, etc. The use of the dynatron for measuring r.f. resistance at still higher frequencies would require a special valve designed with the above considerations in mind.

Acknowledgment

Acknowledgment is due to the University of Durham for permission to publish this work, which was submitted in extended form for the degree of M.Sc.

REFERENCES

[1] Colebrook, F.M.: *Wireless Engineer*, 1931, Vol. 8, p. 581.
[2] Chakravarti, S. P. and Das, P. N.: *Indian Journal of Physics*, 1943, Vol. 17, p. 51.
[3] Fruhauf, H.: *Zeitschrift für Hochfrequenztechnik*, 1931 Vol. 87, p. 229.
[4] Gager, F. M. and Russell, J. B.: *Proc. Inst. Radio Engrs.*, 1935, Vol. 23 p. 1536.
[5] Gavin, M. R.: *Wireless Engineer*, 1939, Vol. 16, p. 287.
[6] Gill, E. W. B. and Morrell, J. H.: *Phil. Mag.*, 1925, Vol. 49, p. 369.
[7] Harvey, A. F.: "High Frequency Thermionic Tubes."
[8] Hay, G. A.: *Wireless World*, 1944, Vol. 50, p. 271.
[9] Hay, G. A: *Electronic Engineering*, 1945, Vol. 17, p. 720.
[10] Iinuma, H.: *Proc. Inst. Radio Engrs.*, 1931, Vol. 19, p. 467.
[11] Meinke, H. H.: *Hochfrequenztechnik und Elektroakustik*, 1937 Vol. 50, p. 50.
[12] Meinke, H. H.: *Hochfrequenztechnik und Elektroakustik*, 1938, Vol. 51, p. 52.
[13] Majewski: *Acta Physica Polonica*, 1939, Vol. 7, p. 340.
[14] Scroggie, M. G.: *Wireless Engineer*, 1933, Vol. 10, p. 527.
[15] Scroggie, M. G.: "Radio Laboratory Handbook."
[16] Terman, F. E.: "Measurements in Radio Engineering."
[17] Tillman, J. R.: *Wireless Engineer*, 1945, Vol. 22, p. 17.
[18] Viti, E.: *Alta Frequenza*, 1938, Vol. 7, p. 536.

TRANSIENT RESPONSE OF FILTERS*

By C. C. Eaglesfield.

(*Mullard Radio Valve Company*).

IN a recent article Tucker[1] has discussed the problem of the transient response of filters. Among other matters he referred to a " 6-element symmetrical " filter section used between resistive terminations equal to its design resistance, and found its analysis intractable.

The object of this note is to demonstrate that a useful solution is easily obtained if the assumption is made that the pass-band of the filter is small compared to the mid-frequency. This is a very likely practical case. The result obtained confirms the correct form of the empirical solution given by Tucker.

For convenience the circuit diagram of the filter and the proportioning of the elements are reproduced from Tucker's article. Thus

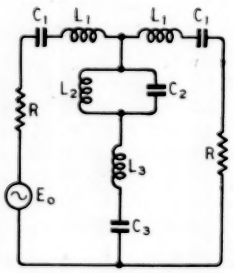


Fig. 1. *6-element symmetrical filter.*

his Fig. 3 becomes Fig. 1 here, and the list of symbols is :—

m = the derivation parameter

n = fractional bandwidth $= \dfrac{\text{actual bandwidth}}{\text{mid-band frequency}}$

R = design impedance

Then

$$L_1 = \frac{m}{n} \cdot \frac{R}{\omega_0} \qquad C_1 = \frac{n}{m} \cdot \frac{1}{R\omega_0}$$

$$L_2 = \frac{n}{2m} \cdot \frac{R}{\omega_0} \qquad C_2 = \frac{2m}{n} \cdot \frac{1}{R\omega_0}$$

$$L_3 = \frac{1 - m^2}{2mn} \cdot \frac{R}{\omega_0} \qquad C_3 = \frac{2mn}{1 - m^2} \cdot \frac{1}{R\omega_0}$$

[1] " Transient Response of Filters," by D. G. Tucker. *Wireless Engineer*, March 1946, Vol. 23, p. 84.

It may be worth remarking that ω_0 is the mid-band angular frequency. The symbol $\omega_1 = n\omega_0$ is also required.

Tucker's equation (15) gives the output voltage (across the right-hand R of Fig. 1) due to the applied E_0 :—

$$\frac{\text{Output volts}}{\frac{1}{2}E_0} = \frac{1 + \dfrac{1 - m^2}{m^2} M^2}{1 + 2M + \dfrac{1 + m^2}{m^2} M^2 + \dfrac{M^3}{m^2}} \quad .. \quad (1)$$

where $M = \dfrac{m}{n}\left(\dfrac{p}{\omega_0} + \dfrac{\omega_0}{p}\right)$ .. .. (2)

and p is the differential operator d/dt.

Equation (1) may be written

$$V = 2 \times \text{Output volts} = \Phi[M(p)] \cdot E_0 \quad .. \quad .. \quad (3)$$

The problem is to evaluate (3) when E_0 is of the form

$$\cos \omega_0 t \, \mathbf{1} \quad .. \quad .. \quad (4)$$

that is, the angular frequency ω_0 modulated by the Heaviside Unit Function ; or to put it in another way, a sudden application of $\cos \omega_0 t$. (The case of the applied angular frequency differing from ω_0 will not be considered.)

Equation (3) becomes :—

$$V = \Phi[M(p)] \cdot e^{j\omega_0 t} \, \mathbf{1} \quad \text{(real part)}$$

$$= e^{j\omega_0 t} \cdot \Phi[M(p + j\omega_0)] \cdot \mathbf{1} \quad \text{(real part)} \quad .. \quad .. \quad (4)$$

This last step is an application of the Heaviside " Shift " Theorem.

We shall find (with the assumption already referred to) that

$$\Phi[M(p + j\omega_0)]$$

is real. Assuming this in advance :—

$$V = \cos \omega_0 t \cdot \Phi[M(p + j\omega_0)] \, \mathbf{1}$$

Thus the envelope of V (call it V_e) is

$$V_e = \Phi[M(p + j\omega_0)] \, \mathbf{1} \quad .. \quad .. \quad (5)$$

The next step is to evaluate $M(p + j\omega_0)$.

* MS. accepted by the Editor, April 1946.

From equation (2) :—

$$M(p) = \frac{m}{n}\frac{(p^2 + \omega_0^2)}{\omega_0 p}$$

Therefore

$$M(p + j\omega_0) = \frac{m}{n}\frac{(p + j\omega_0)^2 + \omega_0^2}{\omega_0 (p + j\omega_0)}$$

$$= \frac{mp}{\omega_1}\frac{p + 2j\omega_0}{p + j\omega_0}$$

$$= \frac{mp}{\omega_1}\frac{2\omega_1 - jnp}{\omega_1 - jnp},$$

since $n\omega_0 = \omega_1$.

$$\rightarrow \frac{2mp}{\omega_1} \text{ as } n \rightarrow 0$$

Thus for the stated condition, that the pass-band is small compared to the mid-frequency, we may write

$$M(p + j\omega_0) = \frac{2mp}{\omega_1}$$

Substitute this in equation (5) :—

$$V_e = \Phi\left[\frac{2mp}{\omega_1}\right]\mathbf{1}$$

Changing the time scale to $t' = \frac{\omega_1 t}{2m}$, we get the simpler expression :—

$$V_e = \Phi(p)\,\mathbf{1} \qquad \ldots \quad \ldots \quad (6)$$

Referring to equation (1) for the form of Φ, it will be seen that we now have a cubic in p in the denominator, instead of a sextic. We may thus expect, and shall in fact find, that equation (6) is easily soluble.

We have

$$\Phi(p) = \frac{1 + \frac{1 - m^2}{m^2}p^2}{1 + 2p + \frac{1 + m^2}{m^2}p^2 + \frac{p^3}{m^2}} = \frac{1 + \frac{1 - m^2}{m^2}p^2}{\left(1 + p\right)\left(1 + p + \frac{1}{m^2}p^2\right)}$$

$$= \frac{1}{1 + p} - \frac{p}{1 + p + \frac{1}{m^2}p^2} = \frac{1}{1+p} - \frac{m^2 p}{(p + \frac{1}{2}m^2)^2 + m^2(1 - \frac{1}{4}m^2)}$$

The solution of (6) is thus the sum of two elementary standard forms, which may be found in any textbook on operational calculus. Reverting to the time variable t, we get

$$V_e = 1 - e^{-a_1 t} - G\,e^{-a_2 t}\sin \omega t \qquad \ldots \quad (7)$$

where

$$a_1 = \frac{\omega_1}{2m}$$

$$a_2 = \tfrac{1}{4}m\omega_1$$

$$G = \frac{m}{\sqrt{1 - \frac{1}{4}m^2}}$$

$$\omega = \frac{\omega_1}{2}\sqrt{1 - \tfrac{1}{4}m^2}.$$

Equation (7) may be compared with Tucker's empirical equation (21) : it will be found to be identical, except that he has not evaluated G.

If numerical values are inserted in equation (7) corresponding to the measured result given in Tucker's Fig. 5 (Curve 1), it will be found that the agreement is not very good : in particular the calculated curve shows about 10 per cent overshoot against the measured 3 per cent. Tucker states that the Q value of his inductors was 100. It may be that this value is not high enough to justify the neglect of the inductor losses ; in this analysis the only resistive elements considered are the terminations. Again it may be that in the measurement the pass-band was not sufficiently small (compared to the mid-frequency) ; the ratio n was 0.037.

One's physical instinct suggests that both the inductor losses and the pass-band were in fact negligible, but it is difficult to think of any other cause of the discrepancy.

BOLOMETERS FOR V.H.F. POWER MEASUREMENT*

By E. M. Hickin, Grad.I.E.E.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England)

SUMMARY.—The increasing use of decimetre and centimetre wavelengths has necessitated some changes in techniques for measuring power. The application of the bolometer method to the measurement of small h.f. powers is described. In this method the power is dissipated in a resistance having a large temperature coefficient (an " indicator ") which forms one arm of a Wheatstone bridge. By direct-current power substitution the indicator may be calibrated and then one measurement of resistance will give the power in the load.

Some details are given of indicators and circuits to deal with powers from a few microwatts to a few watts at frequencies up to 10,000 Mc/s. The limitations of the method and possible sources of error are discussed.

CONTENTS

1. Introduction.
2. Circuits.
3. Indicators.
4. Calibration.
5. Precautions in use.
6. Conclusion.

1. Introduction

THE increasing use of decimetre and centimetre wavelengths introduced a new problem in the measurement of small powers, such as are available at the output of signal generators. The valve voltmeter, often used for this type of measurement at lower frequencies, had reached its limit for use without calibration at the working frequency, and load lamps were neither convenient nor sensitive enough for the small powers involved.

In order to increase the sensitivity and accuracy of measurement of the power in a lamp a property other than its light output can be used. As soon as any power is dissipated in the filament the temperature rises until the radiation and conduction losses equal the power input. This rise in temperature will also cause a change of resistance. If the circuit is so arranged that the resistance of the lamp can be measured then an indication of power will be obtained long before the filament has reached incandescence. Fig. 1 shows the increase in sensitivity obtainable.

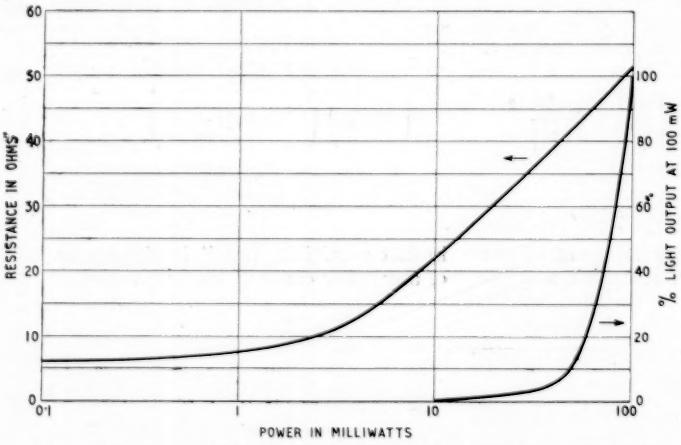


Fig. 1. *Resistance and light output of a tungsten lamp are shown here as functions of the power input. Large resistance changes are obtained well below incandescence.*

The use of this property is by no means new. It was first used to measure heat radiation before the thermopile was developed and it is from this that the name bolometer (Greek: Raymeter) was derived. Tissot, in 1904, described its use for measuring radio-frequency currents, but the thermocouple superseded it. The recent revival in the use of the bolometer has been entirely concerned with the measurement of total power and not of current, and the circuit arrangements

* MS. accepted by the Editor, March 1946.

have, therefore, been somewhat different.

2. Circuits

One method of coupling a load lamp to an oscillator is by placing it across an open line which is itself coupled to the oscillator. This system can be used with slight modification for bolometer measurements as shown in Fig. 2, where a capacitance-shorting bridge

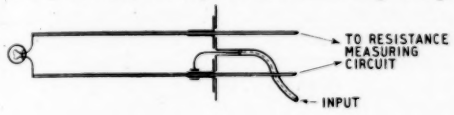


Fig. 2. *A load line can be coupled to an oscillator through an open-wire line.*

is used to enable d.c. leads from the indicator to be taken to a resistance measuring device. A circuit of this type has been used at about 1,000 Mc/s for measuring oscillator outputs and cable attenuations, but the open wire circuit is not so desirable as a concentric system on account of the radiation losses and liability to interference.

Fig. 3 shows a number of circuits for various frequencies; (*a*) and (*b*) are "half-wave" circuits, i.e., the indicator forms a load at one end of the line, with a moveable short-circuiting bridge at the other end and an adjustable tapping point for injecting the input. A capacitance break in the outer conductor enables the necessary d.c. connections to be made. These circuits have been designed for use with a low-impedance indicator (i.e., one which has an impedance equal to or lower than the characteristic impedance of the circuit) and when at higher frequencies the lead length between the filament and the short circuit at the end of the line becomes too great a second bridge must be inserted as in Fig. 3(*c*).

For the most accurate measurement the circuit losses must be reduced to a minimum and for the higher frequencies the sliding contact bridges are not suitable. When the frequency is high enough to enable a waveguide of reasonable dimensions to be used a better type of bridge can be embodied. This is the "non-contact" type shown in Fig. 3(*d*) and is designed to produce so great an impedance discontinuity that the wave is totally reflected before reaching the contact region. Waveguide circuits of the type shown in Fig. 3(*d*) and (*e*) have, with suitable guide and piston dimensions, been used at frequencies greater than 10,000 Mc/s.

3. Indicators

In the circuit shown in Fig. 2 a flashlamp bulb, preferably uncapped, can be used but for frequencies above 600 Mc/s it is preferable to have a "straight-through" construction in which the filament enters and leaves the bulb at opposite ends. This makes the indicator suitable for forming the inner conductor of a concentric line, or for placing across a waveguide.

An indicator, CV95, specially developed for this work is shown in Fig. 4. For ease of construction the filament is first mounted on a mica frame and then sealed into a bulb, and evacuated. It has a 0.01-mm tungsten filament 8 mm in length. As can be seen from Fig. 5, the cold resistance is about 6 ohms and the maximum power which it can dissipate is 120 mW. While higher powers will not cause an immediate burn out, the evaporation of the filament will cause a drift in calibration if this rating is exceeded for long periods.

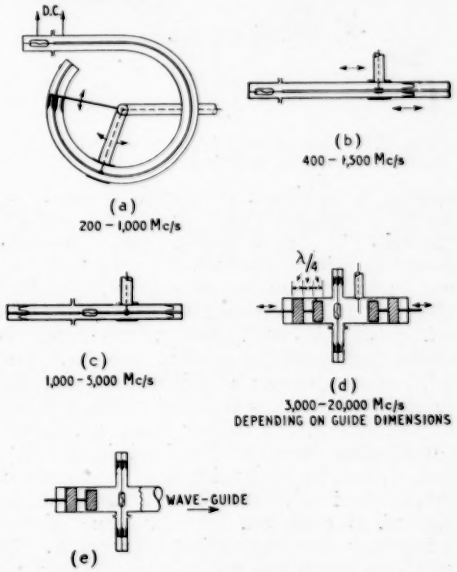


Fig. 3. *Bolometer circuits for various frequency ranges.*

The rating of any given filament can be increased by filling the bulb with an inert gas. The increased conduction loss from the filament will keep the temperature rise down and in this way powers of several watts can be dissipated by the CV95 filament. Fig. 6 shows the result of argon and hydrogen

filling at various pressures (thermal conductivities, Argon 0.00004, hydrogen 0.0003 at 0° C).

To reduce circulating current losses to a minimum the characteristic impedance of the bolometer circuit should equal that of the input line. Similarly the indicator impedance should also equal that of the incoming line, e.g. 75 ohms; but if, as with the CV95, this

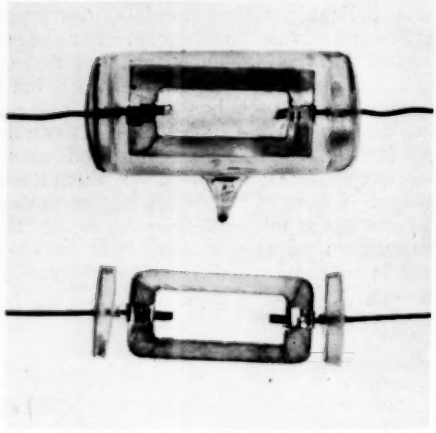


Fig. 4. *CV95 (R3/10) indicator showing construction.*

cannot be achieved it should be made as nearly as possible equal in order to reduce the effect of circuit losses. When a small power is being measured the impedance of the indicator can be increased by adding power from an independent source. If 2-kc/s a.f. power is used the galvanometer is easily decoupled to prevent damage. The a.f. power input can be adjusted to give the temperature, and hence the resistance, required and the r.f. power measured by the substitution method described in Sect. 5 below; i.e., by measuring the change in d.c. power required to rebalance the bridge when the r.f. is removed. The presence of the a.f. power does not affect the result. In Fig. 7 is given the result of measuring the output of an oscillator of constant power (fed through a length of attenuating cable) by a bolometer whose indicator impedance can be varied in the manner described. With no "preheating" the bolometer indicates about 3 mW. at a resistance of 15 ohms. As the resistance is progressively increased by the addition of a.f. power, and the circuit readjusted, the indicated r.f. power also increases, until at 60 ohms about 6 mW is reached. A standing wave detector in the input to the bolometer shows that this effect is not due to mismatching and can therefore be attributed to the circuit losses.

A preferable system for measuring small powers is to have an indicator whose cold impedance is nearer the desired value. One way of obtaining this is by using finer wire; this will also have the advantage of increased sensitivity due to lower thermal capacity. Tungsten cannot easily be drawn below 0.01 mm diameter but platinum can be made with very small diameters by using Wollaston* wire. This consists of a platinum wire coated to some ten times its own diameter with silver. The composite wire is drawn as fine as convenient and the silver is then etched off, leaving the platinum. Wires of 0.003-mm platinum made in this way have been used and an 8-mm length gives a "cold" resistance of about 120 ohms compared with the 6 ohms of the 0.01-mm tungsten.

Higher impedances can also be obtained by using carbon filaments. A 0.06-mm carbon filament of 8-mm length has a cold resistance of 150 ohms dropping to about 90 ohms for an input of 1.5 watts. In addition iron,* by virtue of its permeability,

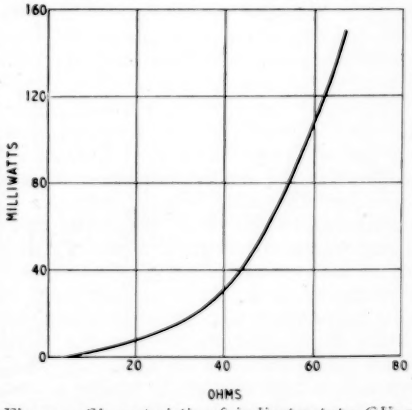


Fig. 5. *Characteristic of indicator type CV95.*

has a higher impedance than non-magnetic metals as the skin effect is more pronounced.

The result of preheating some of these filaments as described above is also shown in Fig. 7.

* The author would like to acknowledge the assistance given by Dr. Chaston (Johnson Matthey Research Dept.) and Mr. D. W. Chappel (N.P.L.) in obtaining Wollaston and 0.01-mm iron wire.

4. Calibration

Calibrations may be made in several ways. If the powers to be measured are greater than a few milliwatts a power-resistance curve may be plotted on d.c. (Fig. 5) and then, using a Wheatstone bridge with low current as shown in Fig. 8(a), the resistance

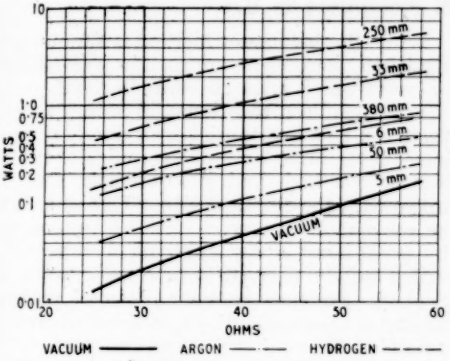


Fig. 6. *Indicator characteristics for various gas pressures* (0.01 *mm tungsten filament.*)

may be measured and hence the r.f. power derived. The reason for not using this method for powers below 1 mW is that the "cold" resistance of the indicator is a function of ambient temperature and calibration will not therefore be reliable unless this is carefully controlled or unless the temperature rises due to r.f. are large compared with local fluctuations. If a CV95 has been calibrated at say 15° C. a rise of only 4° C can give an apparent reading of 100 μW. By placing an identical indicator in the opposite bridge arm compensation for slow changes in ambient temperature can be obtained but the system is not recommended.

The bridge current must also be small so that the d.c. power introduced does not upset the reading. If

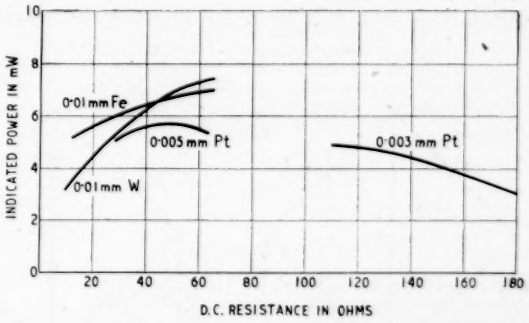


Fig. 7. *The effect of "pre-heating" on the power indication is shown.*

the total bridge current is 0.1 mA the lower limit of power for 1 per cent error is 10 μW when $R_1 = 10R_2$.

When smaller powers have to be measured it is necessary to calibrate the indicator for each reading. This can be simply done by the circuit of Fig. 8(b). The bridge is balanced in the usual way with the r.f. applied and with some arbitrary current I_1 flowing into the bridge. Then the r.f. power is removed and the total bridge current increased until the bridge rebalances. Let the new current be I_2. Since balance has been obtained the indicator resistance, and hence the power in it, must be the same. The difference between the two d.c. powers in the indicator will give the r.f. power.

This power may be obtained in the following way. If I_1 and I_2 are bridge currents (as measured) and i_1 and i_2 the corresponding indicator currents, then :

$$i_1 = I_1\left(\frac{R_1+R_2}{r+\frac{rR_1}{R_2}+R_1+R_2}\right) = I_1\frac{R_2}{r+R_2}$$

similarly $i_2 = I_2\dfrac{R_2}{r+R_2}$

$$\text{R.F. Power} = i_2^2R - i_1^2R$$

$$= (I_2^2 - I_1^2)\left(\frac{R_2}{r+K_2}\right)^2 r$$

$$= (I_2+I_1)(I_2-I_1)\left(\frac{R_2}{r+K_2}\right)^2 r$$

Since $\left(\dfrac{R_2}{r+K_2}\right)^2 r$ will be a constant for a number of readings the sum and difference of two currents is all that will be required. In this way powers down to about 10 μW can be measured.

Another method of measuring small powers has been described elsewhere*; the circuit

* "Radio Measurements in the Decimetre and Centimetre Wavebands," by Clayton, Houldin, Lamont and Willshaw, *Journ. Instn. Elec. Engn.* Pt. III, March 1946.

is shown in Fig. 8(c). A variable high resistor R_4, a microammeter and a switch are added in parallel with the bridge. The bridge is balanced with the switch open before applying the r.f. power. After the application of the r.f. power the switch is closed and R_4 adjusted. The shunting effect will reduce the power supplied to the bridge, and R_4 is reduced until balance is obtained. As in the method of Fig. 8(b) the r.f. power will be the change in d.c. power in the indicator. It can be shown that this power is given by

$$P = r\left(\frac{R_2}{r+R_2}\right)^2 \frac{R_3}{R+R_3} \cdot I_4 \left\{2I_2 - I_4 \frac{R_3+2R}{R_3+R}\right\}$$

where $R = r\dfrac{R_1+R_2}{r+R_2}$, the total bridge resistance.

In the simple case where $r = R_1 = R_2 = R$

$$P = \frac{R}{4}\frac{R_2}{R+R_2}\left[2I_2I_4 - \frac{I_4^2}{2}\frac{R_3+2R}{R_3+R}\right]$$

and if $I_2 \gg I_4$ and R_3 is made $\gg R$,

$P = \dfrac{I_2 I_4 R}{2}$, that is to say $P \propto I_4$ and the microammeter gives a linear indication of power.

This method is very useful for the measurement of powers of the order of 10 μW where I_4 will be of the order of a few microamperes.

Two other methods which can be used for rough measurements in order to reduce calculations to a minimum are to calibrate the out-of-balance current of the bridge in terms of power, or else to measure the voltage across the indicator when it is fed from a battery in series with a high resistance. These methods are useful as output indicators when setting up oscillators or amplifiers.

5. Precautions in Use

While the bolometer method is simple and straightforward there are certain possibilities of errors which must be borne in mind.

1. As has been mentioned in Sect. 3 above there can be a substantial error due to circuit losses if the impedance of the indicator is too low or too high and if insufficient care is taken with the circuit design. This error can be prevented by correct choice of indicator, by preheating, or by applying a correction which can be derived from consideration of Sect. 3. For measurement of small powers sliding contacts should be avoided wherever possible.

2. When the wavelength is comparable with the filament length the distribution of radio-frequency current in the filament will not be uniform. With the CV95 this effect should not be significant at frequencies up to about 5,000 Mc/s. The magnitude and sign of this error will depend on the slope of the power-resistance curve.

3. If the power being measured is amplitude modulated at a low frequency then the resistance of the indicator will pulsate in sympathy with the modulation. Although the pulsation in the bridge current can be smoothed out so that the bridge measures mean resistance, owing to the curvature of

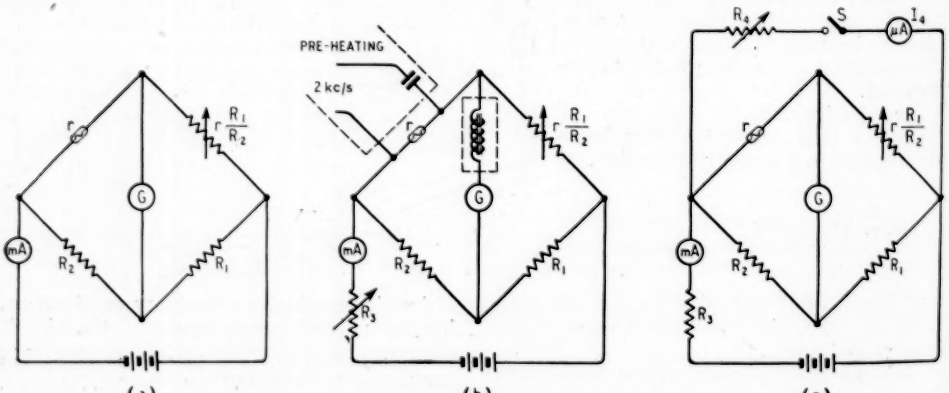


Fig. 8. *Methods of calibrating bolometers for different orders of power.*

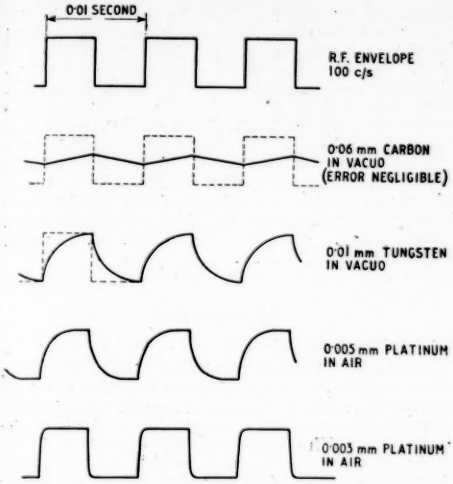

0·01 SECOND
R.F. ENVELOPE 100 c/s
0·06 mm CARBON IN VACUO (ERROR NEGLIGIBLE)
0·01 mm TUNGSTEN IN VACUO
0·005 mm PLATINUM IN AIR
0·003 mm PLATINUM IN AIR

Fig. 9. *Pulsation of resistance with amplitude modulation of carrier.*

the power-resistance relationship, if the pulsation is large the mean resistance will not correspond to the mean power shown by the d.c. calibration. The effect is quite noticeable with the CV95 on short pulses of 50 c/s repetition rate, but soon becomes negligible at higher frequencies of repetition. The time of rise and fall of resistance with pulses of r.f. can be measured by passing a steady current through the indicator and amplifying the voltage pulses generated across it. When these are viewed on a cathode-ray tube the decay time for a CV95 with some 30-mW input has been measured and is about 0·025 second; for 0·005 and 0·003 mm platinum (in air) the times are 0·006 and 0·002 second respectively. Fig. 9 shows the type of picture obtained with various filaments, and it will be seen that the error is least with the filament of highest thermal capacity.

4. It should be mentioned that when thick filaments are used the heat distribution can be non-uniform radially due to skin effect. This effect will be more noticeable when the filament is of a low heat conductivity.

6. Conclusion

Provided proper account is taken of its limitations the bolometer is a valuable tool for power measurement at centimetre and decimetre wavelengths, especially in the range from 100 mW to a few microwatts. The frequency limit has not yet been reached; and for powers of 10 μW and above at frequencies up to 10,000 Mc/s it has provided a good estimate of absolute power and an accurate method for relative measurements.

The work described in this paper was carried out to meet needs arising in various research and development programmes undertaken on behalf of the Admiralty. The author is indebted to the Board of Admiralty for permission to publish this paper.

CORRESPONDENCE

Permeability of Dust Cores

To the Editor, "Wireless Engineer."

SIR,—In the June 1946, issue of *Wireless Engineer*, Burgess gives a formula (cited below) for the computation of the effective permeability μ of a dust core composed of ferro-magnetic particles having an inherent permeability μ_0 and a fractional concentration (by volume) of p. This inspired an investigation under certain hypothetical conditions with the following results.

Hypothesis I. Assuming the iron grains to be identical cubes with corresponding faces parallel, uniformly distributed, and perfectly aligned in rows in three perpendicular directions, I have shown (Appendix A) that for flux perpendicular to the cube faces the effective permeability of the aggregate core is:

$$\mu = 1 + \frac{p}{\frac{\mu_0}{\mu_0 - 1} - \sqrt[3]{p}} \qquad (1)$$

whence, if $\mu_0 >> 1$

$$\mu = 1 + \frac{p}{1 - \sqrt[3]{p}} \qquad (2)$$

This alignment of cubical grains was proposed by Howe, *Wireless Engineer*, January 1933.

Hypothesis II. Assuming the iron grains to be identical cubes with corresponding faces parallel, uniformly distributed but with *random* alignment, I have shown (Appendix B) that:

$$\mu = 1 + \frac{p}{\frac{\mu_0}{\mu_0 - 1} - p} \qquad (3)$$

whence, if $\mu_0 >> 1$

$$\mu_{max} = 1 + \frac{p}{1 - p} \qquad (4)$$

Hypothesis III. Assuming the iron grains to be identical spheres uniformly distributed, Burgess gives the value:

$$\mu = 1 + \frac{3p}{\frac{\mu_0 + 2}{\mu_0 - 1} - p} \qquad (5)$$

whence, if $\mu_0 >> 3$

$$\mu_{max} = 1 + \frac{3p}{1 - p} \qquad (6)$$

The equations (1), (3), and (5) all satisfy the boundary conditions, namely:

If $p = 0$ (no iron) $\mu = 1$
If $p = 1$ (all iron) $\mu = \mu_0$
If $\mu_0 = 1$ $\mu = 1$
If $\mu_0 = \infty$ equations (2), (4), and (6) are obtained.

Assuming sufficiently high values of μ_0 so that equations (2), (4), and (6) are valid, the following table gives the values of μ_{max} attainable with varying concentrations p.

p	Hypothesis I Aligned Cubes (Howe)	Hypothesis II Random Cubes (Lamson)	Hypothesis III Spheres (Burgess)	Legg & Given $\mu_0 = 220$
0.01	1.013	1.010	1.030	1.055
0.02	1.027	1.020	1.061	1.114
0.05	1.079	1.053	1.158	1.310
0.10	1.187	1.111	1.333	1.715
0.20	1.482	1.250	1.750	2.941
0.50	3.424	2.000	4.000	16.83
0.60	4.831	2.500	5.500	25.44
0.70	7.244	3.333	8.000	43.62
*0.74	8.749	3.846	9.538	54.12
0.80	12.16	5.000		74.81
0.90	27.09	10.00		128.3
0.92	34.56	12.50		142.9
0.94	47.05	16.67		159.2
0.96	72.03	25.00		177.3
0.98	147.0	50.00		197.5

* This is the maximum concentration to which spheres may be packed in space.

For any specific value of p, spheres give higher values of μ_{max} than cubical grains of the same volume. Both of the ratios

$$\frac{\mu \text{ (spheres)}}{\mu \text{ (random cubes)}} \text{ and } \frac{\mu \text{ (aligned cubes)}}{\mu \text{ (random cubes)}}$$

increase progressively with p, while the ratio

$$\mu \text{ (spheres)} \,/\, \mu \text{ (aligned cubes)}$$

reaches a maximum of about 1.2 with an iron concentration of about 0.30.

All three hypotheses are based on the assumption that the flux through the medium is in the form of parallel straight lines. In II and III this means that the induction B is the same at every point in the medium. In Hypothesis I the ratio of the cross section of the flux which passes partly in iron to the total flux cross section is $p^{2/3}$, while the induction of the flux which passes wholly in the non-magnetic binder is $1 + \left(\frac{1}{\mu_0} - 1\right) \sqrt[3]{p}$ times the induction in the iron grains. If $\mu_0 >> 1$ this factor becomes $1 - \sqrt[3]{p}$ and independent of μ_0.

In the *Bell System Technical Journal* for July 1940, Legg and Given state a purely empirical equation:

$$\log \mu = p \log \mu_0 \qquad \ldots \quad \ldots \quad (7)$$

which holds over a wide range of p. They describe a molybdenum permalloy powder of assorted grain sizes, 98 per cent being less than 100 microns in diameter (mean diameter 42 microns, r.m.s. diameter 50 microns). Assigning their value of 220 for μ_0 the effective permeabilities given in the last column of the table are obtained.

The very definite excess of empirical results over any of the three hypotheses is, I believe, due to the fact that actual flux lines in the core material are not straight and parallel. We may picture the iron grains as "sucking in" some of the flux in their immediate vicinity so that the induction in the iron grains (and at points in the binder material between two adjacent grains along the major axis of the flux) exceeds the induction at points in the binder between adjacent grains on a line perpendicular to the flux axis. This assumption would raise the values obtained by each of the three hypothetical equations and would become more influential as the concentration p is increased. This explanation is analogous to the analysis of conductivity and lines of current flow in a medium of low conductivity impregnated with particles of higher conductivity.

Cambridge, Massachusetts, U.S.A. H. W. LAMSON.

Appendix A

Derivation of Equation (1)

Let s equal the edge dimension of each iron cube. With the alignment specified, each iron cube would occupy a cubical "assigned space," having an edge dimension w and sides parallel to those of the iron cube. The iron cube is centralized in its assigned space so that it is surrounded on all six faces by laminae of binder material of thickness: $\frac{1}{2}(w-s)$.

Computing the reluctance between opposite faces of this assigned space, there are two flux paths.

Path A, Flux all in binder material

Length of path	w
Cross section of path	$w^2 - s^2$
Permeability	$= 1$
Reluctance	$w/(w^2 - s^2)$

Path B, Flux partly in iron and partly in binder

B1 Through Iron	Length of path	s
	Cross section	s^2
	Permeability	μ_0
	Reluctance	$\frac{1}{\mu_0 s}$
B2 Through Binder	Length of path	$w-s$
	Cross section	s^2
	Permeability	1
	Reluctance	$\frac{w-s}{s^2}$

Since B1 and B2 are in series, the total reluctance of the B path is

$$\frac{s + \mu_0(w-s)}{\mu_0 s^2}$$

Since Paths A and B are in parallel, the total reluctance of the assigned cube of space (a unit in the assembly of any desired core dimensions) becomes

$$\frac{w[s + \mu_0(w-s)]}{w^2 s + w^3\mu_0 - w^2\mu_0 s + s^3(\mu_0 - 1)}$$

Assuming the assigned space to be homogeneous,

Length of flux path	w
Cross section	w^2
Effective permeability	μ
Total reluctance	$1/\mu w$

Equating these two values and solving

$$\mu = \frac{\mu_0 + (\mu_0 - 1)(s^3/w^3 - s/w)}{\mu_0 - (\mu_0 - 1)\, s/w}$$

Then introducing the fractional concentration by volume

$$p = s^3/w^3$$

gives Equation (1).

Appendix B

Derivation of Equation (3)

With uniform but random distribution of the iron grains, any flux line of a given substantial length would traverse the same number of grains.

Consider any tube of flux through the medium of length d and infinitesimal cross section Δ.

Total volume of tube $v_t = \Delta d$

Volume of iron in tube v_i

Whence $p = v_i/v_t$

Length of flux path in iron $= v_i/\Delta$

Length of flux path in binder $= d - v_i/\Delta$
$= (v_t - v_i)/\Delta$

Then the total reluctance to tube of flux will be

$$\frac{v_i}{\mu_0\Delta^2} + \frac{v_t - v_i}{\Delta^2} = \frac{v_i + \mu_0(v_t - v_i)}{\mu_0\Delta^2}$$

Considered as a homogeneous tube of effective permeability μ, the reluctance would be

$$d/\mu\Delta = v_t/\mu\Delta^2$$

Equating these two values and solving

$$\mu = \frac{\mu_0 v_t}{v_i + \mu_0(v_t - v_i)}$$

Introducing p from above leads to Equation (3).

To the Editor, "Wireless Engineer"

SIR,—There has been some discussion of the permeability of iron-dust cores following my paper in the June 1946 issue of *Wireless Engineer* on the "Iron-Cored Loop Receiving Aerial."

The formula derived in the Editorial of January 1933 for the effective permeability of a core of ordered uniform cubical particles was

$$\mu = \frac{\alpha^2\mu_0 + (1 - \alpha^2)}{\mu_0 - \alpha(\mu_0 - 1)}$$

where $\alpha^3 = p$ is the relative volume of the iron. Although this tends to the same form as that for spherical particles when p approaches unity, it has the defect that for zero iron content it gives $\mu = 1/\mu_0$ instead of unity, and for this reason I did not refer to it in my paper.

At large iron densities the usual formula of the Clausius-Mosotti type (equations 1 and 2 of my paper) gives values of μ appreciably smaller than those measured (cf. Legg and Given, *B.S.T.J.*, Vol. 19, p. 385, 1940). This discrepancy may be due to:—

(i) the particles not being spherical,

(ii) the particles being non-uniform in size so that the smaller ones can fit in the spaces between the larger ones,

(iii) the assumption of an unmodified field in the insulator not being valid.

Regarding the latter point, Leigh Page (*Phys. Rev.*, Vol. 60, p. 675, 1941) has postulated that the field in the insulator is reduced to an extent proportional to the magnetic moment of the particles. This effect is determined by a parameter η which increases from zero to unity as the iron content increases.

Page thus found

$$\mu = 1 + \frac{3p(\mu_0 - 1)}{3 + (1 - p)(1 - \eta)(\mu_0 - 1)}$$

and by assuming $\eta = 0.96p^3$ he obtained very close agreement with the experimental data of Legg and Given; the unmodified formula ($\eta = 0$) gives values of μ some five times too small at $p = 0.7$.

It is of interest to note that Page's formula can be written in the form

$$\frac{\mu - 1}{\mu + c} = p\,\frac{\mu_0 - 1}{\mu_0 + c}$$

where $$c = \frac{2 + \eta}{1 - \eta}$$

which is reminiscent of the Clausius-Mosotti formula in which $c = 2$.

R. E. BURGESS.

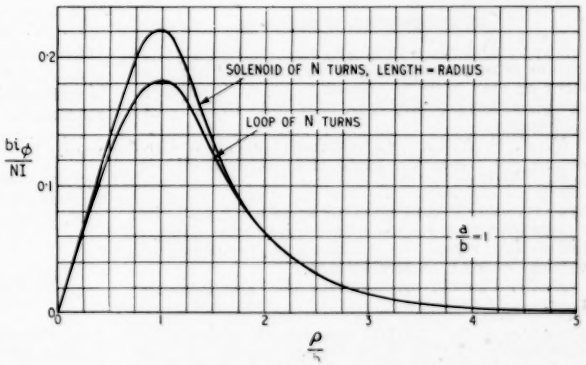
National Physical Laboratory,
Teddington.

Power Loss in Electromagnetic Screens

The Editor, "Wireless Engineer."

Sir,—The formula derived by Mr. C. A. Siocos in your correspondence columns of July last for the eddy current density in a plane sheet, provides for this quantity to be calculated where the energizing inductor has finite length. In order to estimate the error on a calculation which assumed the use of a bunched winding, the eddy current density was evaluated using the formula derived by Mr. Siocos for the case of a solenoid of N turns and carrying a current of I amperes. The length of the latter coil was taken as being equal to the radius and the centre of the coil was considered as being at a distance from the sheet equal to the coil radius. For this case, using the notation of our original article, "Power Loss in Electromagnetic Screens"*,

$$\frac{a}{b} = 1.$$



The current distribution in the sheet is shown in the figure, both for the case of a loop of N turns and for a solenoid of N turns having the above proportions and it will be seen that the density of the screen eddy current at a radius equal to that of the coil is about 1.2 times that which occurs when a loop of N turns is employed. At other radii this ratio

Wireless Engineer, Jan. 1946, p. 8.

becomes less, tending closely to unity for values of ρ/b less than 0.5 or greater than 1.5.

Using the method of the original article, the increase in the resistance of the energizing coil due to Screen No. 2 of Table I of that article was calculated for a solenoid of 55 turns, radius 1 inch, axial length 1 inch and centre of solenoid 1 inch from the plane of the screening wires (i.e. $a/b = 1$). The increase in resistance is found to be 0.207 ohm, while the calculated resistance increase due to a 55 turn loop is 0.166 ohm. It is to be expected, of course, that the ratio between these two values will be somewhat less than $(1.2)^2$.

The agreement between values calculated by the two methods will become better as the ratio of a/b increases but, as the formula derived by Mr. Siocos is no more difficult to use than that originally given, it would be preferable to use the former formula when the energizing inductor is in the form of a solenoid.

C. F. Davidson,
R. C. Looser,
J. C. Simmonds.

London, S.E.5.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

576 948.—Traffic control system, say for an aerodrome, in which microphones are spaced out along the runways and light-up individual gas-discharge tubes grouped in the control room.

Standard Telephones and Cables, Ltd., E. M. S. McWhirter, R. H. Dunn, C. H. Chambers, F. W. Warden and S. S. Hill. Application date 3rd March 1944.

577 080.—Adjustable pick-up device for tracing and locating buried conductors, and for depth-determination.

V. Planer and F. E. Planer. Application date 9th February, 1944.

DIRECTIONAL WIRELESS

576 357.—Direction-finding system, free from polarization errors, comprising a quadrilateral arrangement of four pairs of crossed-loops, each adjusted in phase-quadrature, and coupled to a radio goniometer.

N. F. S. Hecht. Application date (for a secret patent) 3rd November, 1936. *Published 9th March*, 1946.

576 415.—Aerial and reflector system for radiating a blind-landing beam which gives the pilot a distinctive signal to enable him to check his vertical bearing at any instant.

H. M. Dowsett. Application date 4th February, 1942.

576 442.—Aerial and reflector system whereby the distribution of the field-strength within a solid angle gradation can be varied, say for local television purposes, for short-wave working from an aeroplane to ground.

H. M. Dowsett. Application date 27th January, 1941.

576 444.—Elliptical system of reflectors, designed to take the place of the usual feed-line for conveying short-wave energy to a radiating aerial, and so saving copper and dielectric losses.

H. M. Dowsett. Application date 18th April, 1941.

576 555.—Receiver coupled to directive and omni-directional aerials to give a clear-cut "inverse-cardioid" response on a cathode-ray indicator.

N. F. S. Hecht. Application date (for a secret patent) 19th March, 1938. *Published 13th March*, 1946.

576 825.—Reflector and dipole for radiating a hollow beam having a constant radius of maximum electric stress.

H. M. Dowsett. Application date 13th June, 1940.

576 826.—Reflector and dipole for securing a desired concentration of field-strength at a given focus.

H. M. Dowsett. Application date 5th July, 1940.

576 828.—Dipole type of aerial for receiving centimetre waves which have been radiated in the form of a beam having rotational symmetry.

H. M. Dowsett. Application date 22nd August, 1941.

576 941.—Aerial and reflector radiating two concentric conical beams, in order to reduce the time required to spot a target by radiolocation.

H. M. Dowsett. Application date 24th September, 1940.

577 002.—Radio gear for controlling the flight of a bomb fitted with movable ailerons.

B. Thomson. Application date 2nd May, 1940.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

576 358.—Receiver in which double resonator circuits are employed to counterbalance unwanted "noise" or interference with desired signals.

N. F. S. Hecht. Application date (for a secret patent), 13th April, 1938. *Published 13th March*, 1946.

576 469.—Two inter-coupled pentode transitrons for generating square-shaped pulses of a given periodicity at a definite interval after the receipt of a triggering impulse.

The General Electric Co. Ltd. and E. C. Cherry. Application date 23rd October, 1941.

576 672.—Receiver provided with a frequency-discriminating circuit of the rectifier type for offsetting the effect of slight or casual variations in tuning.

The General Electric Co., Ltd. and A. S. Gladwin. Application date 3rd March, 1944.

576 768.—Superhet i.f. transformer, formed with recesses to house the coils, and provided with movable powdered-iron cores to adjust the coupling between the coils.

The British Thomson-Houston Co., Ltd. Convention date (U.S.A.) 5th June, 1943.

576 785.—Permeability tuner in which the powdered-iron core is formed with longitudinal grooves to simulate the effect of a taper, whilst allowing tilt-free movement.

Marconi's W.T. Co., Ltd. (assignees of W. E. Newman). Convention date (U.S.A.) 24th March, 1943.

577 168.—Amplifier circuit, including a velocity modulating tube, in which the impedance presented to the electron beam is made equal to the reciprocal of the mutual conductance of the tube, to improve the signal-to-noise ratio.

C. S. Bull. Application date 11th April, 1941.

577 173.—Mixing circuit, say for a superhet receiver, in which an electron discharge device having a hollow resonator is made to function as a diode rectifier.

C. S. Bull. Application date 21st May and 21st November, 1941.

577 181.—Preparation and composition of a silicon crystal rectifier, containing stabilizing elements of aluminium or beryllium, for use as a mixer in a superhet receiver.

The General Electric Co., Ltd., D. E. Jones, C. E. Ransley, J. W. Ryde and S. V. Williams. Application dates 20th August and 18th October, 1941.

577 191.—System of automatic volume control in a telegraphic receiver, designed to prevent undue strain on the insulation resistance of components of normal quality (addition to 557 225).

Creed and Co., Ltd. and F. P. Mason. Application date 5th January, 1944.

577 201.—Mains-eliminator unit in which the l.t. supply is derived from the h.t. voltage through a mechanical vibrator and a step-down transformer.

The General Electric Co., Ltd., and L. C. Stenning. Application date 19th June, 1942.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

576 622.—Scanning a transparent record, such as a photographic negative, through the medium of a phosphorescent surface for facsimile transmission.

P. D. Zurian. Application date 8th February, 1943.

577 019.—Synchronizing system, suitable for television, in which the regulating-voltage is derived from the frequency-difference in the first stage of the operation, and from the phase-difference in the final or finer stages of adjustment.

"Patelhold" Patentverwertungs, etc., A.G. Convention date (Switzerland) 11th June, 1942.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

576 460.—Power oscillation-generator having a sliding coil in the anode circuit to provide a variable coupling with the tank inductance, so as to match the valve to the load.

Radio Transmission Equipment Ltd., L. Grinstead and K. A. Zandstra. Application date 19th September 1944.

576 656.—Transmitting telegraphic or facsimile signals through a frequency-converting keying-device which is operated instantaneously through a balanced-diode circuit.

F. B. Dehn (communicated by Press Wireless Inc.). Application date 4th September, 1943.

576 685.—Keying circuit for controlling the Morse signal flashes generated by a high-pressure metal-vapour lamp (addition to 508 241.)

The General Electric Co., Ltd., V. J. Francis and E. R. Thomas. Applicztion date 17th March, 1939.

576 719.—Bridge circuit for stabilizing the operation of an oscillation-generator of the polarized vibrating-blade type, used in carrier-wave signalling.

Cie pour la Fabrication des Compteurs and Materiel d'Usines a Gaz. Convention date (France) 11th February, 1941.

576 780.—Reactance-control device for frequency-modulation arranged in the cathode circuit of a valve, and utilizing negative reaction to vary the transconductance of the valve.

Marconi's W.T. Co., Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 25th February, 1943.

577 152.—Construction of composite cable, capable of transmitting mechanical power, and of providing a carrier-wave communication or remote-control channel.

Communications Patents, Ltd., and C. H. Leech. Application date 6th March. 1944.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

576 790.—Systems of pulsed signalling, say for interlocking party-lines on a wired networks, or for supervisory of remote control (addition to 514 991.)

Standard Telephones and Cables, Ltd., E. A. H. Bowsher and H. M. M. D'Assis-Fonseca. Application date 31st March, 1944.

576 804.—Circuit for receiving pulsed signals wherein the impact of the pulse flashes a gas-discharge valve momentarily, thus operating a stepping relay, whilst the cessation of the pulse again flashes the valve to restore all relays.

Standard Telephones and Cables, Ltd. (communicated by International Standard Electric Corporation.) Application date 22nd May, 1944.

576 845.—Pulsed signalling system for remote control, say over a radio link, wherein the duration of a transmitted pulse controls the generation of a series of pulses at the receiving end.

L. H. Drysdale and M. Squires. Application date 8th November, 1943.

577 068.—Cathode-ray tube in which two scanning beams are traversed at different speeds over a common screen, in order to change the time-scale of high-speed events, say for transmitting radio-location data over land-lines.

Marconi's W.T. Co., Ltd., and S. W. H. W. Falloon. Application date 5th October, 1939.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

576 127.—Gradation of the operating potentials applied to the cathode, reflector, and resonator of a discharge tube of the rhumbatron type.

B. J. Mayo. Application date 8th August, 1941.

576 128.—Supporting and anchoring device for the indirectly-heated cathode of a thermionic valve.

The M-O Valve Co. Ltd., E. G. Rowe and J. A. Smyth. Application date 23rd September, 1942.

576 151.—Process for stabilizing the reaction to heat of a tensioned filament, say of the hair-pin type, of a thermionic valve.

The M-O Valve Co. Ltd., D. H. Donaldson and R. O. Jenkins. Application date 22nd January, 1942.

576 369.—Electron-focusing system, comprising permanently-magnetized and apertured electrodes, particularly for an electron microscope.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 31st March, 1943.

576 414.—Electrode arrangement and assembly of a photo-electric cell incorporating heat-resisting baffles of mica, rendered opaque to infra-red rays by pre-firing in hydrogen.

Electrical Research Products Inc. Convention date (U.S.A.) 25th May, 1940.

575 485.—Tuning a magnetron oscillator by the bodily movement of an auxiliary element which is located inside the resonant cavity and is controlled by an externally-heated bi-metallic strip.

" Patelhold " Patentverwertungs, etc. A.G. Convention date (Switzerland) 13th November, 1942.

576 575.—Valve-holder wherein a series of recesses co-operate with a split ring, which is locked in position by lugs engaging with the recesses.

Carr Fastener Co., Ltd., and G. Wagstaff. Application date 20th March, 1944.

576 581.— Valve-holder having axially-spaced abutments which are engaged, above and below, by locking members.

Carr Fastener Co., Ltd., and G. Wagstaff. Application date 27th March, 1944.

576 749.—Mounting the filament of an electron discharge tube on a mandrel of plastic material which is subsequently destroyed by heat.

Westinghouse Electric International Co. Convention date (U.S.A.) 24th April, 1943.

576 883.—Construction of valve-seal designed to prevent the cement from entering the re-entrant " well " and so reducing the insulation-resistance between the electrode leads.

The M-O Valve Co. Ltd. and C. W. Cosgrove. Application date 23rd February, 1944.

577 005.—Velocity-modulating tube in which the " bunching " gap is screened so as to reduce the intensity of the field below that of the second or " working " gap.

Standard Telephones and Cables, Ltd. and J. H. Fremlin. Application date 2nd August, 1940.

577 037.—Oscillation-generator comprising an annular resonator of dumb-bell cross-section, with a continuous transverse gap through which electrons pass radially towards an outer reflecting electrode.

L. F. Broadway. Application date 14th January, 1941.

577 070.—Means for holding a piece of alkaline-earth activating-material inside the helix of a thermionic cathode.

The General Electric Co., Ltd. and B. N. Clack. Application date 18th March, 1942.

577 085.—Arrangement of the field-magnets in a magnetron type of valve to permit of angular adjustment and to facilitate valve-replacement.

D. Jackson and Pye, Ltd. Application date 20th March, 1944.

577 105.—Valve-holder provided with insulating sockets which can " float " to the extent necessary to allow for manufacturing tolerances.

British Solenoids, Ltd. and S. J. Tyrrell. Application date 6th September, 1943.

577 149.—Metal-cased electron discharge tube designed to facilitate the process of evacuation during manufacture.

Marconi's W.T. Co., Ltd. (assignees of C. Herzog). Convention date (U.S.A.) 28th November, 1942.

SUBSIDIARY APPARATUS AND MATERIALS

576 508.—Electro-mechanical selective relay, with centrifugal action, for remote control purposes.

Compania para la Fabricacion de Contadores, etc., and P. Viteau. Application date 2nd March, 1944.

576 631.—Preventing the deterioration of insulation through corona discharges by the use of a coating of coal pulverized to a colloidal fineness.

Westinghouse Electric International Co. Convention date (U.S.A.) 3rd December, 1942.

576 791.—Inductor element suitable for the high-frequency heat-treatment of gear-wheels and like articles formed with re-entrant surfaces.

Standard Telephones and Cables, Ltd. (assignees of V. W. Shannon). Convention date (U.S.A.) 5th April, 1943.

576 817.—Four-pole magnetizing device, arranged to produce two flux-paths at right-angles, for the testing of metal-work for cracks.

The Equipment and Engineering Co., Ltd. and H. B. Swift. Application date 1st September, 1944.

576 903.—Series-parallel arrangement of resistances and reactances coupled to a cathode-ray tube for measuring impedances.

Standard Telephones and Cables, Ltd. (assignees of P. S. Christaldi). Convention date (U.S.A.) 8th June, 1943.

577 022.—Method of cutting and assembling the component parts of a longer-than-normal composite piezo-electric crystal-oscillator.

The Brush Development Co. (assignees of F. Massa). Convention date (U.S.A.) 8th August, 1942.